Public Markets for Claims in Litigation*

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Abstract

We study how public pricing in litigation finance markets affects plaintiffs' financing decisions and settlement bargaining. A plaintiff sells shares of case proceeds via an intermediary platform whose price reveals case quality. Settlement is modeled as a one-sided private information game in which the defendant makes offers. We show that plaintiffs optimally oversell their claims relative to efficient risk sharing. Reduced risk exposure enables credible commitment to tougher settlement demands, and bargaining gains exceed the per-share price discount. More informative prices temper this overselling, as information substitutes for issuance in strengthening the plaintiff's bargaining position. The welfare implications of public prices are nevertheless ambiguous. Better information improves the allocation of risk but sends more disputes to trial. Endogenizing the mass of investors may further reinforce overselling.

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1 Introduction

Litigation finance markets enable litigants to obtain funding from investors for their legal cases, e.g., for corporate litigation and patent infringement disputes. Over the past two decades, these markets have grown substantially, reaching an estimated USD 20 billion annually worldwide by 2025.[1] They attract not only specialized litigation funds but also hedge funds and other institutional investors. While these financial markets may provide better access to justice for individuals and firms, they may also influence the real outcomes of litigation and have thus drawn increasing scrutiny from both practitioners and academics [Beisner and Rubin, 2012, Daughety and Reinganum, 2014].

A central concern is how the structure of financing contracts affects settlement bargaining. The financing contracts typically take a non-recourse structure. Litigants sell shares of future litigation cash flows to investors without repayment obligations if the case fails. According to legal scholars Steinitz [2013] and Solas [2019], these contracts may serve as commitment devices for litigants in the bargaining. By reducing the litigants' exposure to uncertain trial outcomes, they can strengthen the litigants' bargaining positions. With smaller potential losses, they make tougher settlement demands of their opponents. While this may benefit individual litigants, it ultimately drives more cases to court, a potentially costly outcome for the legal system.

A notable recent development in litigation finance, however, has been the emergence of Juristrade, a market exchange launched in 2025 that allows institutional investors to trade shares in litigation claims at publicly observable prices. When a lawsuit is priced publicly via such a market, this fundamentally changes the lawsuit's informational environment. The participants on this public market are often specialized litigation finance funds, which possess data from previously financed cases that are highly informative about the expected proceeds from a claim. Public prices reveal this information to all litigants involved. Since common information usually fosters more agreement in negotiations, one might naively expect such a public market to improve efficiency in legal outcomes, possibly ameliorating the inefficiencies delineated above.

This paper examines the emerging public markets for litigation financing through a theoretical lens and assesses whether their public nature can indeed always improve bargaining efficiency. We construct a parsimonious model that features two channels through which financing affects bargaining: a commitment channel, through selling shares to investors and reducing exposure to risky court outcomes, and an information channel, through the additional information provided by public prices. Crucially, we allow these channels to interact. Litigants choose the shares sold for financing endogenously. Hence, any shift in the informational environment affects bargaining outcomes directly, but also through changes in financing and the concomitant commitment channel. Our model has

[1] See Research Nester [2025].

two stages, financing and settlement bargaining, and each of its assumptions reflects an institutional feature of litigation finance.

Bedi and Marra [2021] highlight risk sharing as a central reason why litigation financing is used. Legal cases are costly and their outcomes highly uncertain, so litigants value the ability to shed part of that risk. A second reason is capital constraints, since a plaintiff may lack the funds to pursue his case. In practice, these constraints are softened by contingency fees, which are often used in conjunction with litigation financing. Financing can nevertheless affect legal strategy and upfront costs, as in Antill and Grenadier [2023]. In our model, we focus on the risk-sharing function. We assume that the investors and the litigants are risk averse and set capital constraints aside.

A recurring concern in the legal literature is that financing pulls the funder's and the plaintiff's interests apart. Steinitz [2013] describes it as follows:

> "[T]he concern is that funders are incentivized to settle early for a discounted but secure amount rather than proceed to a costly trial that may result in a loss or low recovery. [...] And the plaintiff, who no longer bears the risk of a loss, may now have an incentive to resist a reasonable and rational early settlement in favor of a late settlement or even a risky and expensive trial."

The conflict is sharpened by the fact that, in most jurisdictions, the plaintiff remains in control of the settlement bargaining, so the funder cannot simply impose the outcome it prefers. Similar tensions between investors and the party in control are familiar from venture capital. In our model, this conflict arises on its own. The plaintiff sells a share of his claim to the investors on a financial market, and we place no restriction on how well the resulting interests align. Because the plaintiff keeps control of the bargaining, the misalignment gives rise to moral hazard.

Litigation funders are widely regarded as experts in valuing claims, an exercise generally regarded as difficult. Before committing capital, they conduct due diligence, legal analysis, and damages modeling, and as repeat players they accumulate data from previously financed cases. As Solas [2019] observes:

> "Third party funders [...] would be more capable of pricing the value of that claim. Having a more precise assessment [...] levels the playing field and obviously induces more settlements and higher settlement amounts."

Avraham and Wickelgren [2018] corroborate this information advantage, examining the case in which funding agreements are admissible as evidence in court and thus convey the investors' valuation to the parties directly. In our model, this expertise reaches the litigants through the market. Between the financing and the bargaining stage, the price at which the investors trade becomes public, and both litigants learn from it about the likely court outcome.

Most legal disputes are resolved by settlement rather than by trial, and the bargaining takes place under asymmetric information. The defendant naturally knows more about her own conduct, and hence about her liability, than the plaintiff does. Liability is also harder to assess and to prove than the size of the damages. Solas [2019] points to this difficulty as a reason why defendants, who would need to establish their innocence, usually do not obtain financing. Our bargaining stage reflects this asymmetry. Only the defendant knows the share of the damages for which she is liable, while neither party knows how large the damages will turn out to be. The defendant makes a settlement offer, which the plaintiff either accepts or rejects. If he rejects, the case goes to trial, which is costly only in that both risk-averse litigants must bear the uncertain outcome. Among the possible equilibria, we focus on the one in which the defendant's offer reveals her private information. The two stages are linked through the plaintiff. Having observed the public price and sold part of his exposure, he enters the bargaining less sensitive to risk than before.

Public markets for litigation assets are a recent development, and we are the first to study them. However, the insights gained can also apply to syndicated financing arrangements, provided these are verifiable during bargaining with litigation opponents. Our analysis yields five main results.

First, the plaintiff oversells. He sells a strictly larger share of his claim than efficient risk sharing between him and the investor prescribes (Proposition 2). Selling a share of the claim does two things for the plaintiff. It transfers risk to the investor, and it strengthens his position in the settlement bargaining, since a plaintiff with relatively less to lose at trial can credibly demand more from the defendant. At the share that allocates risk efficiently, the gain from further risk transfer is exhausted, so selling slightly more is not costly to the plaintiff. The effect on the bargaining position, by contrast, remains strictly positive, since each additional unit sold reduces his exposure further and raises his settlement demand. The resulting overselling forgoes risk-sharing surplus, lowers the probability that the parties settle, and sends more disputes to trial.

Second, a more informative public price reduces the share the plaintiff sells, provided he does not oversell by too much (Proposition 4). Information and issuance are substitutes in strengthening the plaintiff's bargaining position. A more informative price makes trial less risky for him and raises his settlement demands. Any additional issuance thereby has a smaller effect on the bargaining. However, we require some restriction on overselling. If the plaintiff oversells more, the misalignment with his investor will be larger, and they will appreciate the change in the distribution of equilibrium outcomes from the more informative price differently. For example, the investor will appreciate the potential variance reduction in the outcomes more than the plaintiff. As a consequence, the lower risk penalty in the price may encourage more issuance for severe overselling.

Third, intermediation encourages overselling (Proposition 5). We extend the benchmark by an intermediary platform that attracts a mass of investors at a cost, in exchange for a percentage fee on the financing revenue. The plaintiff benefits from a commitment effect when selling his shares. The intermediary responds to higher issuance by attracting more investors. When the issuance is high, the bargaining outcomes are more risky, which makes the attraction of investors more beneficial. Spreading more risk increases the price, and thereby the revenue share obtained by the intermediary. The plaintiff anticipates this response. His commitment benefit in bargaining continues to increase with more issuance, while the price discount from the increased risk and misalignment is softened by the intermediary. Hence, there is more overselling, relative to a benchmark with the same exogenous mass of investors.

Fourth, financing improves the allocation of risk only within limits (Proposition 6). We compare the aggregate cost of risk borne by the plaintiff, the investor, and the defendant under full retention, private markets, and public markets. Private markets improve on full retention whenever the investor and the defendant are sufficiently risk-tolerant. Whether public markets improve on private ones depends on the informativeness of the price. A more informative price makes each trial less risky and, by the second result, reduces the share sold, both of which improve the allocation of risk. At the same time, it sends more disputes to trial. Holding the public-market issuance fixed, the latter force dominates when the price is sufficiently informative, so public markets allocate risk at least as well as private ones if and only if the price is at most moderately informative.

Fifth, financing raises the incidence of trial. Both private and public markets send more disputes to trial than full retention (Proposition 7). The private comparison is immediate, since any positive share sold lowers the probability of settlement. Under public markets, the more informative price lowers that probability directly. It also reduces the share sold, which works in the opposite direction. However, since the plaintiff still issues a strictly positive share, the direct effect prevails.

These results bear on the regulatory debate. Disclosure initiatives in the United States and the European Union presume that transparency of funding arrangements disciplines the market. In our model its effect is not monotone. More informative prices reduce the plaintiff's overselling, but they also send more disputes to trial, and which force dominates depends on how informative the price already is.

The remainder of the paper proceeds as follows. Section 2 reviews the related literature. Section 3 develops the model, covering settlement bargaining and the financing stage. Section 4 characterizes settlement bargaining as a signaling game and selects the unique separating equilibrium. Section 5 analyzes the financing stage, first in the representative-investor benchmark that isolates the core commitment and price effects (Section 5.1), and then with an intermediary platform that endogenizes the mass of investors (Section 5.2). Section 6 evaluates the welfare consequences along the risk-allocation and legal-activity margins, and Section 7 draws out policy implications and empirical content. All proofs are collected in the Appendix.

2 Related Literature

We contribute to a broad strand in financial economics on how financial contracting and capital structure shape real outcomes in bargaining and renegotiation. This theme appears across several settings. In the workout of distressed corporate debt, credit default swaps can create an "empty creditor" whose hedged exposure strengthens its bargaining position and makes efficient renegotiation harder [Bolton and Oehmke, 2011]. In labor and contract renegotiation, a firm's leverage becomes a strategic device that shifts the surplus it can extract from its counterparties [Perotti and Spier, 1993]. In litigation finance, contracting with an outside investor can affect the litigant's subsequent incentives in settlement bargaining.

Within the literature on litigation finance, our work is most closely related to Daughety and Reinganum [2014] and Spier and Prescott [2019]. Both speak to a central critique of the U.S. Chamber Institute for Legal Reform, that third-party funding makes reasonable settlement offers less attractive and thereby drives greater use of the courts [Beisner and Rubin, 2012]. Studying how financing shapes settlement bargaining and court use, the two nonetheless reach opposing conclusions. Daughety and Reinganum [2014] run counter to the critique, finding that financing promotes settlement and reduces court use, a result driven by the contingent nature of the funding contract. Spier and Prescott [2019] support it, finding that financing depresses settlement, a result driven by the effect of financing on the litigant's effective risk aversion. Neither paper, however, addresses the agency problem between the litigant and the investor, detailed in Steinitz [2013]. The funder prefers an early, discounted but secure settlement, whereas the litigant, shielded from the downside of a loss, holds out for a costly trial. We are the first to propose a model of litigation finance in which this tension arises endogenously.

Daughety and Reinganum [2014] show that an optimal nonrecourse loan, maximizing the joint expected payoff of the litigation funder and the plaintiff, induces full settlement in the bargaining game. This optimal loan fosters agreement by making the plaintiff's expected net recovery from trial independent of his private type, such that all types pool on the same demand. This is different from the ordinary outcome, where private information results in disagreement and court trial. It is noteworthy that, to isolate the effect, the authors assume that the plaintiff's private information is realized only after the financing contract is completed.

Spier and Prescott [2019], by contrast, study contingent settlement contracts between two risk-averse litigants who hold divergent beliefs about the trial outcome, distinguishing contracts written between the parties themselves ("inside") from those written with third-party investors ("outside"). Their central result is the opposite. Because contracting with outsiders lets the parties simultaneously share risk and speculate on their disagreement, writing one lowers the settlement rate and raises the volume and cost of litigation.

Moreover, they show that litigants weakly prefer to contract with competitive risk-neutral outsiders, who bear trial risk more cheaply.

The two papers differ first in how they model the bargaining stage. Daughety and Reinganum resolve settlement through a non-cooperative signaling game, but keep their litigants risk-neutral. Spier and Prescott admit risk aversion, yet reduce bargaining to a reduced-form Nash solution. We combine the two, embedding risk-averse litigants in an explicit non-cooperative bargaining game. This lets us trace how financing affects bargaining through the litigant's risk exposure, and expose the agency conflict between litigant and investor that a reduced-form treatment cannot capture.

The two papers differ again in the financing stage. Daughety and Reinganum let the funder write contingent contracts under symmetric information. Spier and Prescott endow the parties with divergent beliefs and restrict them to simple risk-sharing contracts. To isolate the effect of financing on risk, we follow Spier and Prescott and keep the contract a simple, equity-like claim on the proceeds, abstracting from the contingent waterfall structures of Daughety and Reinganum. While such award-contingent contracts are common in practice, we do not consider them, since we want to explore the economics of the contract's non-recourse nature in the non-contingent case. We then depart from both by adding due diligence on the investor's side, letting us study how the precision of the signal it produces shapes the division of trial risk between litigant and investor.

Our work relates to the literature on the real effects of financial markets, in which information revealed through prices feeds back into real outcomes. Extensive surveys are provided by Bond et al. [2012] and Goldstein [2023]. A similar phenomenon is studied by Avraham and Wickelgren [2018]. Rather than being revealed through a public price, the investor's information is conveyed by admitting the private funding contract as evidence in court. They model funders who receive informative binary signals about case strength and compete under monopoly and Bertrand duopoly, and ask whether such admissibility is welfare-improving. There the admissible contract credibly signals case quality to the court, as a high-signal funder must offer more favorable terms to separate from a low-signal imitator, reducing funders' monopoly rents and expanding access to financing. In our setting, the public price instead signals case quality to the defendant, and we study its effect on settlement bargaining.

More recent work has examined litigation funding from complementary perspectives. Antill and Grenadier [2023] model bargaining as a dynamic real option and consider the effect of relaxing the plaintiff's ex-ante financial constraints on the defendant's defensive spending. They show that financing can raise joint surplus. A better-funded plaintiff deters the defendant from wasteful defensive spending, an argument most applicable to the US legal system, where initial investment capacity for legal strategies is crucial. Unlike ours, their model abstracts from asymmetric information to isolate the effect of the relaxation of the financial constraints and the dynamic real option bargaining.

Our findings also have some empirical foundation. In particular, Lera et al. [2023] find, among other results, that litigation financing prolongs trials. Drawing on a dataset of 2.51 million U.S. civil lawsuits terminated between 1977 and 2020, they document wide variation in case duration and win rates across case types and circuits. They relate these outcomes to financing and find that it broadens access to justice by letting risk-averse law firms pursue high-risk cases, but at the cost of greater aggregate litigation spending and heavier court caseloads. They reason that their results are indicative of litigation finance affecting outcomes through risk-sharing. This accords with our own mechanism, in which financing, by shielding the litigant from volatile judgments, reduces settlement and sends more disputes to costly trial.

Other noteworthy contributions in the field of litigation finance include the following. Grenadier and Grenadier [2026] show how disclosure requirements regarding financing arrangements impact litigation outcomes. Choi and Spier [2018] investigate interactions of legal cases with stock markets, where a short position can be taken on the opponent in litigation. Landeo and Nikitin [2018] consider how attorney financing shapes litigation incentives in related settings. Dana and Spier [1993] examine contingent fee arrangements when attorneys possess superior information about case quality relative to clients.

The most closely related works from the classical litigation and settlement bargaining literature include Bebchuk [1984] and Schweizer [1989]. Bebchuk analyzes one-sided incomplete information where defendants hold private information about trial outcomes, showing that asymmetric information can prevent settlement and lead to trial. Schweizer extends this to two-sided incomplete information, demonstrating that refinement criteria select a unique separating equilibrium with positive litigation probability. Our model builds on this foundation with uncertain damages and a defendant who privately knows the share of those damages for which she is liable. This two-dimensional structure allows us to cleanly separate the two informational forces at work, the bargaining breakdown arising from the defendant's private information about her liability share and the information about the damages conveyed by the public financing price.

For the bargaining stage with incomplete information, we model a signaling game in which the privately informed defendant makes offers to the plaintiff. This generates a multiplicity of perfect Bayesian equilibria. We rely on established selection criteria to consider the effects of financing on the most plausible equilibrium outcome in the bargaining stage. For equilibrium selection, we employ the D1 criterion [Banks and Sobel, 1987, Cho and Sobel, 1990], which selects a unique equilibrium outcome. Myerson [1984] develops a neutral bargaining solution specific to bargaining under incomplete information. Applied to our setting it yields very similar results, though with additional complexities in the resulting payoff expressions. We discuss the robustness of our results to the choice of the bargaining protocol at the end of Section 4. For broader surveys of bargaining under incomplete information, see Ausubel et al. [2002].

3 Model

A plaintiff holds a litigation claim against a defendant, which may be resolved through settlement or trial. Before bargaining, the plaintiff can sell a share of future case proceeds to an investor, who publicly reveals case quality through the price. We develop a two-stage model, first financing and investor due diligence, then settlement bargaining under incomplete information. The bargaining is static and kept as simple as possible.

3.1 Settlement bargaining

Litigants and types. A plaintiff P (he) holds a claim against a defendant D (she), which a court may adjudicate at trial. The defendant's type is the share of the damages for which she is liable, $\theta_D \in \{\theta_g, \theta_b\} \subset (0, 1)$ with $\theta_g < \theta_b$. At trial the court orders her to pay the fraction θ_D of the realized damages J. We call the low-liability type θ_g the good type and the high-liability type θ_b the bad type. Both parties share a common prior $\psi = \Pr(\theta_D = \theta_g)$. The defendant privately observes θ_D, whereas the plaintiff does not. The gross damages J are uncertain to both parties, who share a common, diffuse (improper) prior over J. Their beliefs about the award are pinned down by the due-diligence signal introduced below.

Actions and utilities. The defendant proposes a transfer $x \in \mathbb{R}$ to the plaintiff, who chooses to accept with some probability $\pi \in [0, 1]$. In case of acceptance, the defendant pays x to the plaintiff. Otherwise, the case proceeds to trial. Both litigants exhibit constant absolute risk aversion with $u_j(w) = -e^{-\rho_j w}$ for $j \in \{P, D\}$ and $\rho_j > 0$. Under settlement with transfer x, the plaintiff receives $u_P(x)$ and the defendant receives $u_D(-x)$. Under trial, payoffs are $u_P(\theta_D J)$ and $u_D(-\theta_D J)$. Trial imposes no direct costs. The only cost of litigation is the uncertainty about the award $\theta_D J$ that both parties bear, arising from the common damages risk J and, for the plaintiff, from his uncertainty about the liability share θ_D. This reflects the assumption that the plaintiff bears no cost of handling the case, which is motivated by the fact that the plaintiff's lawyers manage the case.

Equilibrium concept. We analyze perfect Bayesian equilibria (PBE) of the settlement bargaining game. A PBE consists of a strategy profile and a belief system satisfying sequential rationality and belief consistency. Let $\hat{\psi}(x)$ denote the plaintiff's posterior probability that the defendant is the good type, $\Pr(\theta_D = \theta_g \mid x)$, upon observing offer x. For equilibrium selection, we refine the set of perfect Bayesian equilibria using the D1 criterion [Banks and Sobel, 1987, Cho and Sobel, 1990].

3.2 Financing

In anticipation of the future cash flows from the litigation, the plaintiff sells part of his claim. We first analyze a benchmark case with a representative investor who purchases the plaintiff's claim directly. We then extend the analysis to a litigation finance intermediary platform that intermediates between the plaintiff and a continuum of investors.

Representative investor. Before bargaining, the plaintiff can sell a share $\gamma \in [0,1]$ of future case proceeds to a representative investor I (it). The proceeds include both settlement payments and court awards. If the plaintiff sells share γ at price p per share, he receives γp upfront and retains the fraction $(1-\gamma)$ of the eventual case outcome. The investor exhibits constant absolute risk aversion $u_I(w) = -e^{-\rho_I w}, \quad \rho_I > 0$.

The financing stage generates information about case quality in two steps. The investor performs due diligence, evaluating the plaintiff's case and producing a signal $s = J + \varepsilon$ about the gross damages. Under the diffuse prior, observing s induces the common posterior belief

$$J \mid s \sim \mathcal{N}\big(\mu_s, \, \sigma_s^2\big), \qquad \mu_s = s,$$

where σ_s^2 is the residual variance about the award left after the investigation. For simplicity, we refer to its reciprocal $\tau \equiv \frac{1}{\sigma_s^2}$ as the informativeness of the due diligence, reflecting the investor's investigative capacity. A more accurate investigation (higher τ, lower σ_s) leaves less residual uncertainty about the award while holding the posterior mean $\mu_s = s$ fixed. We assume that the investor is uninformed about θ_D, as liability is inherently harder to evaluate and prove than the size of the potential award.[2] After observing s, the plaintiff chooses his financing share γ, and the investor sets a price p per unit. The defendant does not observe s directly but can infer it from the publicly observable price p, which is determined by the investor's valuation conditional on signal s and observable share γ.

Platform financing. We extend the benchmark to a litigation finance intermediary platform F that intermediates between the plaintiff and a continuum of prospective investors $i \in \mathcal{I}$. Let $m \in \mathcal{M} = \mathbb{R}_+$ denote the mass of investors the platform attracts into the deal. Intermediary platforms face costs in soliciting participation, onboarding investors, deploying information technology infrastructure, and facilitating the sharing of full confidential case details under non-disclosure agreements. We model these frictions through a cost function

$$\kappa : \mathcal{M} \to \mathbb{R}_+, \quad \kappa'(m) > 0.$$

[2] Solas [2019] mentions this difficulty in assessing liability as a reason why defendants usually do not obtain financing.

We allow for both convex and concave cost structures, capturing economies or diseconomies of scale in investor acquisition. The platform is compensated for its intermediation through a percentage fee $\phi \in (0, 1)$ levied on the financing revenue. When the plaintiff sells the share γ at the per-unit price p, the platform collects $\phi\,\gamma p$ and the plaintiff receives the remainder $(1 - \phi)\,\gamma p$. We treat the fee as a parameter of the platform's business model throughout, and bound it below the level made precise in Section 5.2.

Before any capital is committed, the platform conducts due diligence on the case and forms a signal s of its quality, which it transmits to the plaintiff. Observing s, the plaintiff decides how large a stake γ to sell. Observing the issuance, the platform determines the mass m of investors to attract at cost $\kappa(m)$. Participating investors receive the full case details under non-disclosure agreements, which motivate the cost of attraction. To share risk optimally, the share γ is distributed equally among the m participating investors, so each investor i acquires exposure $q = \gamma/m$, representing its individual stake in the litigation outcome. The resulting market price p is publicly observable, allowing the defendant also to infer s.

Timeline. The game proceeds as follows:

(i) *Types.* Nature draws the liability share $\theta_D \in \{\theta_g, \theta_b\}$ with prior ψ and fixes the gross damages J, on which both parties hold a diffuse prior. The defendant observes θ_D. Neither party observes J.

(ii) *Financing.* The financing process generates signal $s = J + \varepsilon$, which induces the posterior $J \mid s \sim \mathcal{N}(\mu_s, \sigma_s^2)$. Observing s, the plaintiff chooses the share γ to sell.

(iii) *Investor attraction.* Observing (s, γ), the platform attracts a mass m of investors into the deal at cost $\kappa(m)$.

(iv) *Pricing.* Observing (s, γ, m), investors price the asset at p. The price p, together with γ and m, is publicly observable and allows the defendant to infer the signal s. The platform retains the fee $\phi\,\gamma p$, and the plaintiff receives $(1 - \phi)\,\gamma p$.

(v) *Bargaining.* Observing (p, γ, m), the defendant infers s and bargains over settlement with the plaintiff, with offer x and acceptance probability π. In case of rejection, a trial transfers $\theta_D J$.

The representative-investor benchmark is the degenerate case $m = 1$, in which stage (iii) is trivial, the single investor's own due diligence generates s, and no intermediation fee is charged, $\phi = 0$.

4 Settlement Bargaining

We proceed by backward induction and characterize the settlement bargaining outcome for a given financing choice γ and signal s. The bargaining stage is a signaling game in which the defendant proposes a settlement transfer to the plaintiff. Like most signaling games, the stage exhibits equilibrium multiplicity. We will select a unique equilibrium using the D1 criterion [Banks and Sobel, 1987, Cho and Sobel, 1990].

For notational ease, we define the certainty equivalent transfers at which the plaintiff and defendant are indifferent between settlement and trial for a given type θ. Conditional on type, the trial award θJ is normal, $\theta J \sim \mathcal{N}(\theta\mu_s, \theta^2\sigma_s^2)$. Selling the share γ reduces the plaintiff's exposure to the case outcome, so that after financing his effective risk aversion is $(1-\gamma)\rho_P$. Under constant absolute risk aversion, the certainty equivalents take the closed form

$$\bar{x}_{P,\theta} \;=\; \theta\mu_s - \tfrac{1}{2}(1-\gamma)\rho_P\,\theta^2\sigma_s^2, \qquad \bar{x}_{D,\theta} \;=\; \theta\mu_s + \tfrac{1}{2}\rho_D\,\theta^2\sigma_s^2.$$

These are the plaintiff's separating minimum acceptable offer and the defendant's maximum acceptable offer, respectively. The defendant's certainty equivalent is increasing in θ, so $\bar{x}_{D,b} > \bar{x}_{D,g}$. A larger liability share raises both the expected award and the risk premium the defendant is willing to pay to avoid trial. The plaintiff's certainty equivalent trades off a higher mean against a higher variance, so $\bar{x}_{P,b} > \bar{x}_{P,g}$ holds exactly when the expected award dominates the risk premium. The ordering depends on the financing share γ, an equilibrium object. The case where $\gamma = 0$ is the most binding one. The following assumption implies this ordering for this configuration and consequently for any financing share.

Assumption 1. *The expected award dominates the risk premium,* $\mu_s > \frac{1}{2}\rho_P\,\sigma_s^2\,(\theta_g + \theta_b)$.

We further restrict the relative magnitudes of the players' risk aversion. The litigants are typically individuals or firms exposed to a single case, and they are smaller and less liquid than the funds that invest in litigation claims. Hence, we make the following assumption.

Assumption 2. *The litigants are more risk averse than the investors,* $\min\{\rho_P, \rho_D\} > \rho_I$.

The bargaining stage is affected by this assumption through the plaintiff's effective risk aversion $(1-\gamma)\rho_P$, which is taken as given in this section and depends on the investors' risk aversion through financing. Because selling shares reduces the plaintiff's exposure, financing will imply the ordering $(1-\gamma)\rho_P < \rho_D$.

Separating equilibria. Each defendant type makes a distinct offer, denoted by x_g and x_b respectively. Upon observing the offer, the plaintiff fully infers the defendant's type, i.e., it holds that $\hat{\psi}(x_g) = 1$ and $\hat{\psi}(x_b) = 0$. Separation is achieved through the plaintiff's acceptance rule. While the bad type defendant's offer is accepted with certainty $\pi_b = 1$, the good type's lower offer is accepted only with interior probability $\pi_g \in (0, 1)$. This imposes a signaling cost through the risk of trial in case of rejection. If, conversely, both transfers were accepted with certainty, both types would pool on the less costly offer and separation would fail.

The separating equilibrium offers are pinned down by the plaintiff's indifference between settlement and trial conditional on the inferred type. The bad type defendant chooses offer $x_b = \bar{x}_{P,b}$, which is the smallest transfer the plaintiff accepts with certainty. The good type offers $x_g = \bar{x}_{P,g}$, which makes the plaintiff indifferent between accepting and rejecting. The acceptance probability π_g is bounded by the incentive constraints of both types. Trial must be likely enough to deter the bad type from mimicking the cheaper offer x_g, which requires $u_D\big(-\bar{x}_{P,b}\big) \geq \pi_g \, u_D\big(-\bar{x}_{P,g}\big) + (1 - \pi_g) \, u_D\big(-\bar{x}_{D,b}\big)$. Rearranging this constraint yields an upper bound on the acceptance probability,

$$\pi_g \;\leq\; \frac{e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D \bar{x}_{P,b}}}{e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D \bar{x}_{P,g}}} \;\equiv\; \bar{\pi}_g.$$

The good type's incentive constraint provides a lower bound $\underline{\pi}_g$. Under too much trial risk, the good type would prefer the bad type's certainly accepted offer. Hence, any equilibrium acceptance probability must lie within $\pi_g \in [\underline{\pi}_g, \bar{\pi}_g]$. Off-path beliefs are chosen to maximally deter any deviations. The plaintiff attributes any deviations to the bad type, specifically $\hat{\psi} = 0$, everywhere.

The D1 criterion uniquely selects the separating equilibrium with acceptance probability $\bar{\pi}_g$. Intuitively, the criterion requires the plaintiff to attribute any off-path offer to the defendant type most willing to deviate to it in the respective equilibrium. Formally, the criterion compares the sets of plaintiff responses under which each type is willing to deviate. Due to the monotonicity of our model, this reduces to a comparison between types of the lowest acceptance probability making deviations profitable for a given deviation offer and equilibrium payoff. For the selection proofs, we use the fact that the good type's lower and less risky trial payment makes her threshold on the acceptance probability steeper in the deviation offer, and adjust for the equilibrium payoffs. Equilibria with acceptance probability $\pi_g < \bar{\pi}_g$ are eliminated as follows. Consider an off-path offer slightly above $\bar{x}_{P,g}$. By continuity, the bad type's indifference threshold for the acceptance probability lies above $\bar{\pi}_g$, whereas the good type's lies below. D1 therefore requires the off-path belief $\hat{\psi} = 1$, which makes the deviation profitable. The separating equilibrium with acceptance probability $\bar{\pi}_g$, by contrast, survives. At $\bar{\pi}_g$ the bad type's incentive constraint binds, so in the neighborhood of $\bar{x}_{P,g}$ the types' thresholds on the acceptance probability converge. For offers just above, the flatter bad-type threshold lies below the good type's, making the bad type more willing to deviate. D1 therefore sets $\hat{\psi} = 0$ on $(\bar{x}_{P,g}, \bar{x}_{P,b})$.

Pooling and semi-separating equilibria. In a pooling equilibrium, both defendant types make the same offer x_{pool}. The plaintiff's posterior upon observing this offer is equal to the prior belief. Acceptance with positive probability requires the transfer to reach the plaintiff's pooled reservation $\bar{x}_{P,\psi}$, while the good type is willing to settle only up to $\bar{x}_{D,g}$, so pooling equilibria exist precisely when $\bar{x}_{P,\psi} \leq \bar{x}_{D,g}$ and are supported by off-path beliefs attributing any deviation to the bad type. Under the D1 criterion, all pooling equilibria are eliminated. We focus here on the case with acceptance with certainty, where both types obtain the same equilibrium payoff. Any downward deviation above $\bar{x}_{P,g}$ is more profitable for the good type, due to the difference in size and riskiness of the trial. D1 therefore requires the off-path belief $\hat{\psi} = 1$, which makes the deviation profitable.

In a semi-separating equilibrium, both types make a common offer with positive probability. There cannot be more than one such offer, since otherwise both types would have to be indifferent across several offers simultaneously, and the indifference conditions cannot hold jointly. Any common offer reproduces the payoff structure of pooling, so the argument that eliminates pooling applies. No semi-separating equilibrium survives D1.

Proposition 1. *The unique equilibrium of the settlement bargaining game surviving D1 is separating.*

- *The bad type defendant offers $x_b = \bar{x}_{P,b}$. The plaintiff accepts with probability 1.*
- *The good type defendant offers $x_g = \bar{x}_{P,g}$. The plaintiff accepts with probability*

$$\pi_g = \frac{e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D \bar{x}_{P,b}}}{e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D \bar{x}_{P,g}}}.$$

Off-path beliefs assign probability 1 to the bad type.

The proof can be found in Appendix A.

The selected equilibrium also satisfies Pareto optimality within the set of separating equilibria, a common criterion in the bargaining literature. Proposition 1 characterizes the bargaining outcomes for a fixed financing share γ. Before turning to the financing stage, we need to understand how the financing affects these bargaining outcomes. The following lemma records these comparative statics.

Lemma 1. *Both of the defendant's equilibrium offers are strictly increasing in the financing share γ, whereas the plaintiff's acceptance probability is strictly decreasing:*

$$\partial_\gamma x_b > 0, \qquad \partial_\gamma x_g > 0, \qquad \partial_\gamma \pi_g < 0.$$

Moreover, the bad type's offer rises strictly faster in γ than the good type's offer, $\partial_\gamma \bar{x}_{P,b} - \partial_\gamma \bar{x}_{P,g} > 0$. The plaintiff's acceptance probability is also concave in the financing share, i.e., $\partial^2_\gamma \pi_g < 0$.

The proof can be found in Appendix A.

The defendant's equilibrium offers increase in the financing share γ. Selling a larger share lowers the plaintiff's effective risk aversion $(1-\gamma)\rho_P$, and thereby increases the plaintiff's certainty-equivalent value of the risky trial outcome and, with it, the equilibrium offers. Intuitively, a less risk-averse plaintiff demands a higher settlement from the defendant. The acceptance probability π_g decreases in the financing share γ. Recall that it is set by the bad type's binding incentive constraint. The acceptance probability is just low enough that the bad type prefers separating at $\bar{x}_{P,b}$ to mimicking $\bar{x}_{P,g}$. A higher $\bar{x}_{P,b}$ requires a lower π_g, while a higher $\bar{x}_{P,g}$ requires a higher one. Thus, the sign of $\partial_\gamma \pi_g$ is determined by which offer rises faster in the financing share. Due to the higher risk involved in the higher trial payment share for the bad type, the bad type's offer rises faster, and π_g thus falls unconditionally. Intuitively, a less risk-averse plaintiff is more willing to take a case to risky trial.

This outcome is in line with the legal literature described in the introduction. The property $\partial_\gamma \pi_g < 0$ identifies a legal friction induced by financing. For all financing shares $\gamma > 0$, the share of cases brought to court is higher than without financing. The effect does not depend on the magnitude of the share sold. It arises whenever the plaintiff sells any positive stake. Trial is a costly mode of resolution. It consumes resources that settlement conserves, so settlement is the efficient way to resolve a dispute. In the model as specified, the only cost of a trial is the exposure to outcome risk. In reality, there are certainly additional costs of trial, which we discuss in Section 6 on welfare.

Robustness of the bargaining protocol. The settlement stage assigns the entire proposal power to the defendant, who makes a take-it-or-leave-it offer. This protocol is chosen for tractability, and it appears to be the most natural set-up for a wholly non-cooperative game; it is not essential for the results. The channel through which financing operates is the plaintiff's disagreement value: selling a share of the claim lowers his effective risk aversion and raises his certainty equivalent of trial. Any protocol in which the settlement terms improve for a litigant whose disagreement value rises transmits this shift in the same direction. In particular, the comparative statics of the offers in Lemma 1 are preserved under generalized Nash bargaining, in which the transfer divides the settlement surplus over the trial certainty equivalents according to fixed weights. Nash bargaining, however, admits no rejection in equilibrium, so it is silent on the frequency of trial. The neutral bargaining solution of Myerson [1984], which extends the cooperative approach to bargaining under incomplete information, delivers the same comparative statics also for the trial outcome. The resulting payoff expressions are more involved, and the levels of the equilibrium offers differ, but the direction of the effects of financing on settlement terms and on the frequency of trial is unchanged.

5 Analysis: Financing

5.1 Representative Investor

We first analyze the benchmark case where a single representative investor purchases the plaintiff's claim. This setting isolates the core economic forces without the complications of investor aggregation. The results derived here serve as building blocks for the platform analysis in Section 5.2. Both this benchmark and the platform extension use the uniquely selected separating equilibrium. We take the prior to be interior, $\psi \in (0, 1)$, so that both liability types arise with positive probability. This is necessary for separation. Degenerate ψ makes the defendant's type common knowledge, and there is nothing to separate.

Pricing. The representative investor evaluates the litigation asset after observing the signal s and the plaintiff's financing choice γ. Let $Y(\gamma)$ denote the case proceeds per unit of the claim from the separating equilibrium bargaining continuation game, with equilibrium payoffs determined by the equilibrium selected in Section 4,

$$Y(\gamma) = \begin{cases} \bar{x}_{P,\theta}(\gamma), & \text{if settlement occurs,} \\ \theta J, & \text{if the case goes to trial,} \end{cases}$$

where $\bar{x}_{P,\theta}(\gamma)$ are the equilibrium settlement transfers from Proposition 1; settlement occurs with probability 1 for θ_b and with probability $\pi_g(\gamma)$ for θ_g. The investor purchases share γ at price p per unit and exhibits constant absolute risk aversion ρ_I. Competitive pricing implies the investor is indifferent between purchasing and the outside option, which we normalize to zero, with

$$p(s, \gamma) = -\frac{1}{\rho_I \gamma} \log \mathbb{E}_s \left[e^{-\rho_I \gamma Y(\gamma)} \right].$$

This certainty-equivalent price decreases in γ. As the investor absorbs a larger share, its risk exposure increases, depressing the price it is willing to pay per unit. The price also depends on the signal s through the posterior distribution of J.

Definition 1. *The* risk parity *share* $\gamma_{\text{rep}}^{\text{parity}}$ *is the share at which the plaintiff and investor bear equal risk-adjusted exposure,*

$$\rho_P(1 - \gamma_{\text{rep}}^{\text{parity}}) = \rho_I \gamma_{\text{rep}}^{\text{parity}},$$

which solves to $\gamma_{\text{rep}}^{\text{parity}} = \rho_P/(\rho_P + \rho_I)$.

The risk parity share is an important reference point for welfare. It is the share that optimally allocates risk between the two parties and thereby aligns the plaintiff's preferences with those of the investor. At $\gamma_{\text{rep}}^{\text{parity}}$, the plaintiff's effective risk aversion on his retained stake $(1-\gamma_{\text{rep}}^{\text{parity}})$ equals the investor's risk aversion on its purchased stake $\gamma_{\text{rep}}^{\text{parity}}$, so both parties hold identical preferences over settlement versus trial, eliminating any misalignment regarding subsequent bargaining strategies. This is the classical Borch rule for Pareto-optimal risk sharing among risk-averse agents, under which optimal shares are inversely proportional to risk aversion [Borch, 1962].

Optimal financing. Given the signal s, the plaintiff maximizes his expected utility from the financing and bargaining outcomes, choosing the share γ of future cash flows to sell:

$$\max_{\gamma\in[0,1]} \ \mathbb{E}_s\left[-e^{-\rho_P(\gamma p+(1-\gamma)Y(\gamma))}\right] \quad \text{with} \quad p=p(s,\gamma).$$

The first term in the exponent captures revenue, reflecting how much the plaintiff values the gains from the competitive certainty-equivalent price. The second term in the exponent captures the plaintiff's utility from the retained stake. Taking the certainty equivalent of this objective, the plaintiff equivalently maximizes

$$\max_{\gamma\in[0,1]} \ U_P(\gamma) \ \equiv \ -\frac{1}{\rho_I}\log\mathbb{E}_s\left[e^{-\rho_I\gamma Y(\gamma)}\right]-\frac{1}{\rho_P}\log\mathbb{E}_s\left[e^{-\rho_P(1-\gamma)Y(\gamma)}\right].$$

The financing share γ reaches the plaintiff through two channels, the allocation of trial risk between plaintiff and investor, and the plaintiff's bargaining posture toward the defendant. The risk-allocation channel is governed by the Borch rule. At risk parity the allocation of trial risk is efficient, so the joint risk-sharing surplus from reallocating risk is maximized, conditional on holding the bargaining outcome fixed. Any departure from $\gamma_{\text{rep}}^{\text{parity}}$ is a loss of surplus. Beyond parity, the discount at which the investor accepts an additional share exceeds the plaintiff's valuation of the risk transferred. The bargaining channel is governed by a commitment effect, which favors the plaintiff and transfers surplus from the defendant. Selling part of the case makes the plaintiff effectively less risk-averse on his remaining shares. By Lemma 1, he bargains more aggressively, demanding more in the separating settlements $\bar{x}_{P,\theta}(\gamma)$ from the defendant. The tougher bargaining naturally also decreases the plaintiff's acceptance probability π_g for the good type.

Taking the derivative of $U_P(\gamma)$, we combine an exponential tilting argument with some adjustments for the change in the underlying probability distribution, when the acceptance probability π_g changes with the shares sold γ. Define the two risk exposures $a_I(\gamma)\equiv\rho_I\gamma$ and $a_P(\gamma)\equiv\rho_P(1-\gamma)$, and for any random variable Z and exposure t the tilted expectation $\tilde{\mathbb{E}}_t[Z]\equiv\frac{\mathbb{E}_s[Ze^{-tY}]}{\mathbb{E}_s[e^{-tY}]}$, that is, the expectation under the risk-adjusted measure of a party with exposure t. Differentiating gives

$$U_P'(\gamma) = \underbrace{\tilde{\mathbb{E}}_{a_I}[Y] - \tilde{\mathbb{E}}_{a_P}[Y]}_{\text{risk allocation}} + \underbrace{\gamma\,\tilde{\mathbb{E}}_{a_I}[\partial_\gamma Y] + (1-\gamma)\,\tilde{\mathbb{E}}_{a_P}[\partial_\gamma Y]}_{\text{bargaining commitment}} - \underbrace{\Big(\frac{1}{\rho_I} h_{a_I}(\gamma) + \frac{1}{\rho_P} h_{a_P}(\gamma)\Big)}_{\text{acceptance probability}},$$

where the effect of the change in the acceptance probability on party j's valuation is

$$h_{a_j}(\gamma) \equiv \psi\,\partial_\gamma \pi_g(\gamma)\,\frac{e^{-a_j \bar{x}_{P,g}(\gamma)} - \mathbb{E}_s\big[e^{-a_j \theta_g J}\big]}{\mathbb{E}_s\big[e^{-a_j Y}\big]}, \qquad j \in \{I, P\}.$$

The bracket in the numerator compares the good type's settlement demand with its trial branch, both priced at the exposure a_j. It vanishes at $a_j = a_P$, where $\bar{x}_{P,g}$ is by construction the plaintiff's own certainty equivalent of that trial, and is negative above it. Since $\partial_\gamma \pi_g < 0$, the term h_{a_j} is therefore negative for exposures below the plaintiff's own and positive for exposures above it. In particular $h_{a_P} \equiv 0$ at every share, while h_{a_I} changes sign at risk parity, being negative below and positive above. At parity the risk-allocation and h_{a_j} terms vanish, so the derivative collapses to the strictly positive bargaining-commitment term. Below parity it stays positive, as risk sharing, commitment, and the remaining margins all improve as γ rises. Signing the derivative this way yields the following result.

Proposition 2. *Consider the representative-investor benchmark in the separating equilibrium and fix a signal s. Then the plaintiff's optimal financing share $\gamma^{\text{eq}}_{\text{rep}}(\tau)$ is unique, satisfying*

$$\gamma^{\text{parity}}_{\text{rep}} < \gamma^{\text{eq}}_{\text{rep}}(\tau) \le 1.$$

The proof can be found in Appendix A.

Overselling verifies the ordering assumed in Section 4. At the risk-parity share, the plaintiff's effective risk aversion on the retained stake equals the investor's risk aversion on the purchased stake, $(1-\gamma)\rho_P = \gamma\rho_I < \rho_I$. Since the equilibrium share exceeds parity, Assumption 2 yields $(1-\gamma^{\text{eq}}_{\text{rep}})\rho_P < \rho_I < \rho_D$: the plaintiff enters the bargaining less risk averse than the defendant, as assumed.

The plaintiff prefers overselling to risk parity. At parity the bargaining commitment channel dominates the risk-allocation channel, so the sale of an additional unit is strictly profitable to the plaintiff. The reason lies in the orders at which the two channels operate. The risk-allocation penalty is second order, because parity maximizes the risk-sharing surplus, so the surplus is flat there and a marginal sale beyond $\gamma^{\text{parity}}_{\text{rep}}$ costs the plaintiff only at second order. The bargaining gain, by contrast, is first order, since hardening the plaintiff's posture raises the separating demands, and hence the transfer from the defendant, at a first-order rate. The remaining margin, the effect that runs through the acceptance probability π_g, vanishes at parity, because there the good type's offer is exactly the plaintiff's certainty equivalent of trial under the same risk exposure, resulting in indifference and making the marginal effect through π_g second-order. Hence, the first-order bargaining gain dominates, resulting in overselling relative to risk parity. Underselling is dominated as well, since below parity raising γ moves the allocation toward the Borch optimum and strengthens commitment at the same time, so both channels point in the same direction. The optimum thus lies strictly above parity, $\gamma_{\text{rep}}^{\text{eq}}(\tau) > \gamma_{\text{rep}}^{\text{parity}}$.

The main non-financial consequence of overselling is that it further raises the use of the courts. The plaintiff sells beyond parity in order to harden his bargaining posture, which lowers the good type's acceptance probability π_g. An even larger fraction of disputes relative to risk parity therefore fails to settle and is carried to trial. Relative to the parity allocation, overselling thus both forgoes risk-sharing surplus and sends more disputes to court.

The result does not necessarily depend on the plaintiff obtaining a competitive price from the investor. Overselling hardens the plaintiff's posture and so raises the value of the underlying claim. The force operates on the size of the asset, not on the division of surplus in the financing stage. A similar distortion would therefore arise if the investor held the market power and set both the price and the allocation. The result, however, does rest on the non-contractibility of the plaintiff's acceptance strategy in bargaining. From a purely financial perspective, contractibility is naturally desirable. It would separate ownership from control, lead to commitment for the bargaining stage, and resolve the moral hazard. In our model, economic ownership is the only available commitment technology, which is second-best from the perspective of an investor with market power. While contractibility would resolve the allocation of risk, the effect on the bargaining would not be obvious. We leave this avenue to future work, as our present equilibrium concept may not accommodate contracts written on bargaining behavior.

The assumption of non-contractibility in our model is however not made for analytical convenience but reflects an institutional fact. In the United States, and in many other jurisdictions, authority over whether and on what terms to settle rests with the client as a matter of professional-conduct law, while the doctrines of champerty and maintenance and the standard prohibition on funder control bar investors from directing the litigation or its settlement [Solas, 2019]. Funding agreements are accordingly written as passive, non-recourse claims on the proceeds. The investor buys cash flows, not decision rights. The plaintiff's retention of bargaining control, which we maintain throughout, thus mirrors the law as it stands, and it is this separation of financial exposure from bargaining authority that turns the financing share into the plaintiff's only available commitment technology.

Full sale. The distortion is most pronounced at its limit. Selling the entire claim leaves the plaintiff with no financial exposure but undiminished bargaining authority, so the separation of risk-bearing from control is complete. Such full assignment is not merely theoretical. While many jurisdictions cap or prohibit the outright sale of a claim, others permit the plaintiff to assign it in full, so that $\gamma = 1$ is a feasible and enforceable contract. It is therefore of interest to ask under which conditions the plaintiff's privately optimal share reaches this corner.

Proposition 3. *There exists a unique threshold $\underline{\rho}_I$ such that for all $\rho_I \leq \underline{\rho}_I$, the plaintiff optimally chooses full sale of his case, i.e., $\gamma^{\text{eq}}_{\text{rep}}(\tau) = 1$.*

The proof can be found in Appendix A.

The logic is the same as in Proposition 2, now evaluated at the corner. There the plaintiff weighs the gain from selling the final unit of his claim against the price concession this last sale requires, and that concession depends on how the investor prices the risk it absorbs. When the investor is close to risk-neutral, it takes on the whole claim at essentially no discount, so the per-unit price no longer falls as the plaintiff sells more. The risk-allocation cost of overselling vanishes, and only the first-order bargaining gain from a tougher posture remains. The plaintiff then strictly prefers to sell everything. By continuity the same conclusion holds for any sufficiently risk-tolerant investor, which is what the threshold $\underline{\rho}_I$ delimits. As the investor becomes more risk-averse, taking the entire claim onto its book requires a steeper reduction in price per unit sold. Equivalently, the investor's risk aversion can be read as a lack of liquidity. A risk-tolerant investor absorbs quantity with little effect on the price, while a risk-averse investor is a thin market in which size moves the price against the seller. Past the threshold this penalty overturns the commitment gain and returns the optimum to the interior.

Price informativeness effects. Evidence from litigation practice presented in the introduction suggests that obtaining funding from a litigation finance fund or hedge fund serves as a signal of case quality. Defendants interpret such backing as an indication that sophisticated investors, who conduct extensive due diligence before committing capital, have vetted the claim and found it meritorious. Conversely, plaintiffs view successful fundraising as external validation of their case's strength, reducing their own uncertainty about trial prospects. This dual signaling role raises a natural question. Does the informativeness of the public price from the due diligence reduce the plaintiff's incentive to derisk through selling shares? In order to answer this question, we need to consider the comparative statics of the bargaining outcomes in the informativeness of the public price.

Lemma 2. *The plaintiff's settlement certainty equivalents respond to the informativeness of the public price, τ, according to $\partial_\tau \bar{x}_{P,\theta} = \frac{a_P \theta^2}{2\tau^2} \geq 0$ and $\partial_\tau \partial_\gamma \bar{x}_{P,\theta} = -\frac{\rho_P \theta^2}{2\tau^2} \leq 0$ for $\theta \in \{\theta_g, \theta_b\}$. The good type's acceptance probability is strictly decreasing in the informativeness of the price, $\partial_\tau \pi_g < 0$. It is strictly concave, $\partial^2_\tau \pi_g < 0$, whenever the informativeness does not exceed the threshold $\bar{\tau}_\pi \equiv \frac{1}{4}\rho_D\big(\rho_D + (1-\gamma)\rho_P\big)\theta_b^2$.*

The proof can be found in Appendix A.

The lemma separates the three channels through which the informativeness of the public price shapes the financing decision. The first is the direct effect on the offers. A higher τ makes trial less risky, so the risk-averse plaintiff values a certain settlement less and his separating demand $\bar{x}_{P,\theta}$ rises. Less uncertainty allows the plaintiff to demand more. The second is the cross-effect through the amount of shares sold. More information about the trial and less exposure to the trial are natural substitutes. More technically, since $\partial_\tau \partial_\gamma \bar{x}_{P,\theta} = -\rho_P \theta^2 / 2\tau^2 \leq 0$, the sensitivity of the settlement demand to the share sold is itself decreasing in price informativeness. Hence, the commitment value of overselling is smaller precisely when the price reveals more about the expected trial outcome. The third effect is the indirect equilibrium force the increasing offers impose on the good type's acceptance probability. A more informative price turns out to reduce acceptance. The reason is that this force inherits the first channel. As trial grows less risky, the plaintiff raises his separating demand, and a tougher demand is one that is accepted less often, so more of these disputes are carried to trial.

The conclusion that greater informativeness raises the use of the courts is not fully robust to our modeling choices. In our setting the trial imposes no cost on the plaintiff beyond the risk it carries, so a tougher separating demand raises the transfer he can extract with no offsetting penalty when the case fails to settle. Suppose instead that the plaintiff bears a court cost whenever a dispute proceeds to trial. A more informative price then raises the rejection rate, and the higher expected court cost works against his incentive to raise the demand as trial grows less risky. The channel we describe survives only as long as this cost stays below a certain level, beyond which the plaintiff moderates his separating demand and the sign of the effect on court use need no longer hold. The result should accordingly be read as accurate up to a bounded litigation cost rather than as unconditional.

To evaluate the effects of public prices, we consider the interaction between the informativeness of the price and the issuance by the plaintiff. Since the plaintiff trades off the revenue from selling to the utility from retention, the effect of greater price informativeness on issuance reduces to a comparison of who values the induced changes in the distribution of the underlying asset Y more. We recall that the preferences of the plaintiff and the investor are not aligned due to overselling. Hence, higher expected value is valued more by the plaintiff, whereas lower variance is valued more by the investor. One might naively conjecture that additional information reduces the variance of the claim and is therefore valued most by the investor, so that issuance rises. This intuition, however, turns out to be incorrect whenever the overselling is moderate. One countervailing effect is shown in the Lemma above. Greater informativeness of the public price raises the rejection rate, which can ultimately offset the risk reduction on the trial branch.

The comparison turns on the wedge between the two parties' exposures to the claim, which is the gap between their effective risk aversions at the chosen share,

$$\Delta_a \;\equiv\; a_I - a_P \;=\; \rho_I\,\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau) - \rho_P\big(1-\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau)\big) \;=\; (\rho_P+\rho_I)\big(\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau) - \gamma^{\mathrm{parity}}_{\mathrm{rep}}\big).$$

The gap measures how far the share sold lies above risk parity and is therefore strictly positive when the plaintiff oversells. We find the following result.

Proposition 4. *There exists a threshold $\bar{\Delta}_a > 0$ such that at any interior optimum at which the gap between the two parties' effective risk aversions falls below it, $\Delta_a < \bar{\Delta}_a$, a more informative public price decreases issuance,*

$$\partial\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau)/\partial\tau < 0.$$

The proof can be found in Appendix A.

To see why this must hold, recall the plaintiff's first-order condition for the equilibrium issuance. It has three parts, the bargaining commitment, the acceptance probability, and the allocation of risk. Consider the effect of price informativeness on each part in turn. The first channel runs through the commitment value of selling, the force that drives the plaintiff past risk parity. Parting with a share hardens his bargaining posture and raises the separating demands $\bar{x}_{P,\theta}$. By the cross-partial of Lemma 2, $\partial_\tau\partial_\gamma\bar{x}_{P,\theta} < 0$, the price's informativeness τ and the share sold γ are substitutes in sustaining this commitment. Each unit sold buys less commitment the more informative the price, so a more informative price erodes the incentive to oversell. The second channel runs through the good type's acceptance probability. By Lemma 2, a more informative price lowers π_g and thereby carries more disputes to the risky trial, while at the same time reducing the risk of each trial, so the change in the total variance is ambiguous. The plaintiff's valuation of the marginal share is untouched, since he is indifferent between settling and trial, but the investor is not indifferent. The sign of this channel is therefore ambiguous. The third channel runs through the allocation of risk between the two parties, and its sign is ambiguous as well, because a more informative price acts on the risk of the claim in two opposing ways. It compresses the risk on each trial, and that reduction is worth more to the more risk-averse investor than to the plaintiff, which tilts him toward selling. It also widens the spread between the two types' settlement demands, since the risk premium the plaintiff concedes grows with the liability share at stake, and this raises the risk of the claim and tilts him toward retaining.

Whether the direction of the commitment channel can be overturned depends on how far the plaintiff sells above risk parity. The risk-allocation channel operates only on the wedge between the two parties' exposures to the claim, which is the gap Δ_a defined above, and so does the residual part of the acceptance channel through which the investor's discount for trial is priced. At the parity share the plaintiff and the investor price risk identically, both terms therefore equal zero and only the negative commitment channel remains. Away from parity each of the other two terms is bounded by the gap we constructed. The endogenous threshold $\bar{\Delta}_a$ is the ratio, evaluated at the candidate optimum, of the combined strength of the unambiguous commitment channel to the largest possible version of the jointly ambiguous terms. It is a sufficient-statistic bound and restricts neither liability share.

5.2 The Intermediary Platform

To understand whether and how an intermediary platform can affect the financing of legal claims, specifically the amount of issuance, we generalize the model by endogenizing the investor mass. The representative-investor benchmark of Section 5.1 can be viewed as an extreme case of this environment, one in which attracting additional investors is prohibitively costly. In this subsection we relax that restriction. Recall from Section 3.2 that the plaintiff first chooses the share γ to sell and that the platform F, observing the issuance, attracts a mass m of investors at cost $\kappa(m)$, distributing the issued share equally among them, so that each investor carries the individual exposure $q = \gamma/m$. The representative-investor benchmark corresponds to $m = 1$. The platform is compensated for this intermediation through the percentage fee ϕ introduced in Section 3.2. It collects the fraction ϕ of the financing revenue γp and bears the syndication cost, so its profit from the deal is $\phi\,\gamma p - \kappa(m)$, while the plaintiff receives the remainder $(1-\phi)\,\gamma p$ of the revenue.

Before solving the model, we motivate why we represent the market behind the platform as a mass of risk-averse investors, and why both ingredients, the mass and the risk aversion, are needed. The reference point is the noisy rational expectations equilibrium of Admati [1985], in which a large population of risk-averse agents trades claims to uncertain payoffs. The equilibrium price aggregates the participants' diverse information, but reveals it only partially, since noise prevents full revelation. Every participant is therefore left bearing residual payoff risk, the price contains a discount that compensates for this residual risk, and the size of the discount is governed by the aggregate risk tolerance of the population absorbing the supply. Our financing stage is a reduced form of such a market. The signal produced by due diligence and revealed through the public price stands in for the information the market aggregates, and the mass of participating investors, whose risk tolerances add up across members [Wilson, 1968], stands in for the market's capacity to bear the risk that remains. The residual uncertainty about the award plays the part of the noise, and it must remain strictly positive for the reduced form to be coherent. If the information revealed from the market resolved the award, the investors would carry no risk, the per-unit price would carry no discount and would not respond to the mass, and neither the plaintiff's demand for risk transfer nor the platform's costly recruitment of additional risk bearers would have an object.

Pricing. Pricing parallels the representative-investor benchmark, applied investor by investor. After observing (s, γ, m), each investor purchases the quantity $q = \gamma/m$ of the claim $Y(\gamma)$ at the price p per unit. Competitive pricing makes each investor indifferent between purchasing and its outside option,

$$p(s, \gamma, m) = -\frac{1}{\rho_I q} \log \mathbb{E}_s\left[e^{-\rho_I q Y(\gamma)}\right] = -\frac{m}{\rho_I \gamma} \log \mathbb{E}_s\left[e^{-\frac{\rho_I}{m}\gamma Y(\gamma)}\right].$$

As before, the price falls in the exposure each investor absorbs. But that exposure is now γ/m rather than γ, so at any fixed issuance the price rises in the mass of investors. The second expression makes the aggregation explicit. The pool of investors prices the claim exactly as a single investor with risk aversion ρ_I/m would, since risk tolerances add up across its members [Wilson, 1968]. We call ρ_I/m the market's *effective risk aversion*. The Borch rule then yields the risk-parity share of a market of mass m,

$$\gamma_{\text{pl}}^{\text{parity}}(m) = \frac{\rho_P}{\rho_P + \rho_I/m},$$

which is strictly increasing in m. It holds that $\gamma_{\text{pl}}^{\text{parity}}(1) = \gamma_{\text{rep}}^{\text{parity}}$ and $\gamma_{\text{pl}}^{\text{parity}}(m) \to 1$ as the market becomes fully liquid with $m \to \infty$.

A fixed-mass benchmark. To discern the effect of the platform on financing, we consider a non-strategic benchmark, similar to the representative investor case, but with an exogenous investor mass m. In this case, the plaintiff's certainty-equivalent objective retains the structure of the representative investor benchmark. Selling the share γ at the market price yields the revenue $\gamma\, p(s, \gamma, m)$, so

$$U_P(\gamma; m) \;\equiv\; -\frac{m}{\rho_I} \log \mathbb{E}_s\left[e^{-\frac{\rho_I}{m}\gamma Y(\gamma)}\right] - \frac{1}{\rho_P} \log \mathbb{E}_s\left[e^{-\rho_P(1-\gamma)Y(\gamma)}\right],$$

which is the representative-investor objective with ρ_I replaced by the effective risk aversion ρ_I/m. Under the fee the plaintiff collects only the fraction $1 - \phi$ of the revenue, so his fixed-mass objective is

$$U_P^{\phi}(\gamma; m) \;\equiv\; U_P(\gamma; m) - \phi\, \gamma\, p(s, \gamma, m),$$

and we write $\gamma_{\text{fix}}(m)$ for its maximizer. The analysis of Section 5.1 therefore carries over market by market upon this substitution, provided the fee is small enough in the sense made precise in the appendix. The plaintiff then oversells relative to the parity of his market, $\gamma_{\text{pl}}^{\text{parity}}(m) < \gamma_{\text{fix}}(m) \leq 1$, and the optimum reaches the full-sale corner once the market's effective risk aversion falls below the full-sale threshold, $\rho_I/m < \underline{\rho}_I$. At

$m = 1$ and $\phi = 0$ we recover the representative-investor benchmark. Under the timeline of Section 3.2, however, $\gamma_{\text{fix}}(m)$ is not the problem the plaintiff solves in equilibrium. He issues before the platform attracts investors, so the mass he faces is not a parameter but an object his own issuance moves.

Optimal investor attraction. We solve the financing stage backwards and begin with the platform, which observes the signal and the issuance and chooses the mass of investors it attracts. The best response is accordingly a function $m(\gamma)$ for every signal, and we suppress the dependence on s throughout. The platform collects the fee on the revenue its intermediation generates and bears the syndication cost,

$$\max_{m \in \mathcal{M}} \; \Pi(\gamma; m) \;=\; \phi\,\gamma\,p(s,\gamma,m) - \kappa(m).$$

Only the revenue depends on the mass, and it does so exactly as the plaintiff's utility does, since the retained stake is priced independently of m, so that $\partial_m(\gamma p) = \partial_m U_P$. We can write the partial derivative of the plaintiff's expected utility with respect to the investor mass rearranged in terms of tilted expectations, with the definition of the spread exposure $a_m \equiv \frac{\rho_I}{m}\gamma$, as

$$\partial_m U_P(\gamma; m) \;=\; \frac{\gamma}{m}\Big(p(s,\gamma,m) - \tilde{\mathbb{E}}_{a_m}[Y]\Big) \;=\; \frac{1}{\rho_I}\int_0^{a_m} t\,\tilde{\mathrm{Var}}_t\big(Y(\gamma)\big)\,dt \;>\; 0,$$

where the second equality is established in the appendix. The first expression compares an average with a margin. Any additional investor mass improves the price at which the entire issuance γ clears, an inframarginal gain the plaintiff collects on every unit already sold, while the marginal investor it recruits values the claim only at the tilted expectation $\tilde{\mathbb{E}}_{a_m}[Y]$. The gain is therefore positive because the price the whole issue earns exceeds this marginal valuation. It is scaled by γ/m, the exposure each new investor relieves. Observe further that revenue is strictly concave in the mass, and as $m \to \infty$ the per-investor exposure vanishes and the marginal value of an additional investor tends to zero. On the other hand, the marginal cost does not, since $\kappa' > 0$, so the optimal mass is finite. With weakly convex syndication costs the best response is therefore unique, characterized by the first-order condition $\phi\,\partial_m U_P(\gamma; m) = \kappa'(m)$. The fee scales the marginal value of depth, so the platform's best response coincides with the one a surplus-maximizing intermediary would choose at the inflated cost κ/ϕ.

The financing stage is solved by composing these definitions in the order of the timeline. The price $p(s,\gamma,m)$ is the competitive price, the mass rule $m(\gamma)$ is the platform's best response, maximizing $\Pi(\gamma;\,\cdot\,)$ for every γ, and the issuance $\gamma_{\text{pl}}^{\text{eq}}$ maximizes the plaintiff's utility once this rule is folded in, $U_P^{\phi}\big(\gamma; m(\gamma)\big)$. We write $m_{\text{pl}}^{\text{eq}} \equiv m(\gamma_{\text{pl}}^{\text{eq}})$ for the resulting mass. Since no private information is revealed within this stage, this backward composition is simply subgame perfection. We also make one assumption to add structure to the problem.

Assumption 3. *The syndication cost is weakly convex and twice continuously differentiable, and sufficiently small that the platform's best response is interior and satisfies $m(\gamma) \geq \rho_I/\rho_P$ for every $\gamma \in [\frac{1}{2}, 1]$.*

The assumption asks the syndicate to reach risk parity with the plaintiff on the issuance range used below. A mass $m \geq \rho_I/\rho_P$ of investors has aggregate risk tolerance $m/\rho_I \geq 1/\rho_P$, so the market bears risk at least as well as the plaintiff it insures. This aligns the two versions of the model. At parity the plaintiff's efficient share is at least one half, $m\rho_P/(m\rho_P + \rho_I) \geq \frac{1}{2}$. The conditions on the cost itself are regularity. Weak convexity makes the best response unique, and twice continuous differentiability makes it differentiable, which the proof of Proposition 5 uses to sign its slope.

Assumption 4. *The expected award dominates the risk premium in the stronger sense $\theta_g\mu_s \geq \rho_P\sigma_s^2\big(\frac{1}{2}\theta_g^2 + \frac{1}{4}\theta_b^2\big)$.*

We impose this condition for tractability. It strengthens Assumption 1, with which it coincides at $\theta_b = 2\theta_g$, and it keeps the certainty equivalent of the plaintiff's retained stake weakly decreasing in the issuance, which the proof of Proposition 5 uses to sign the marginal value of issuance.

The fee enters the plaintiff's issuance trade-off, since he keeps only the fraction $1 - \phi$ of the marginal revenue. The result below requires it to be small. The bound arises as follows. Up to the parity of any market the plaintiff may face, his no-fee marginal value of issuance is strictly positive, while the revenue the fee taxes has bounded slope. The ratio $\partial_\gamma U_P/\partial_\gamma(\gamma p)$ therefore has a strictly positive infimum $\bar{\phi}$, which is the bound the proof constructs. For any fee below $\bar{\phi}$, every fixed-mass optimum lies above the parity of its market, and the equilibrium issuance remains weakly above that parity. The bound can be interpreted based on a fixed ratio κ/ϕ, which pins down the mass the platform attracts.

The effect of an endogenous investor mass $m_{\text{pl}}^{\text{eq}}$ on the equilibrium issuance $\gamma_{\text{pl}}^{\text{eq}}$ is twofold. On the one hand, there is a level effect. A larger mass of investors naturally encourages the plaintiff to sell more. On the other, there is a strategic effect at the margin. The plaintiff benefits from an additional commitment effect. When the plaintiff commits to sell more, the intermediary will attract more investors in the subsequent stage, as a response to the higher per-investor exposure and risk from trial. Comparing $\gamma_{\text{pl}}^{\text{eq}}$ with the single-investor optimum would conflate the two effects. The fixed-mass optimum $\gamma_{\text{fix}}(m_{\text{pl}}^{\text{eq}})$ absorbs the level effect. It credits the plaintiff with the whole risk-spreading advantage of $m_{\text{pl}}^{\text{eq}}$ investors, and so nets out everything responsiveness does not. Whatever issuance exceeds $\gamma_{\text{fix}}(m_{\text{pl}}^{\text{eq}})$ must therefore be strategic. It can arise only because the plaintiff anticipates that the market itself responds to what he sells. This comparison is what the remainder of the section makes precise, and we build it by solving the two stages of the financing game in turn.

Proposition 5. *Fix a signal s. Let the pair* $(\gamma_{\mathrm{pl}}^{\mathrm{eq}}, m_{\mathrm{pl}}^{\mathrm{eq}})$ *characterize the equilibrium issuance and investor mass. Consider a fee* ϕ *small enough that the plaintiff still oversells at the fixed-mass benchmark,* $\gamma_{\mathrm{fix}}(m_{\mathrm{pl}}^{\mathrm{eq}}) \geq \gamma_{\mathrm{pl}}^{\mathrm{parity}}(m_{\mathrm{pl}}^{\mathrm{eq}})$*. For* $\gamma_{\mathrm{fix}}(m_{\mathrm{pl}}^{\mathrm{eq}}) < 1$*, the plaintiff sells strictly beyond his fixed-mass optimum,*

$$\gamma_{\mathrm{pl}}^{\mathrm{eq}} \;>\; \gamma_{\mathrm{fix}}(m_{\mathrm{pl}}^{\mathrm{eq}}).$$

The proof can be found in Appendix A.

By Lemma 1, overselling raises the settlement demand, lowers the good type's acceptance probability, and thereby raises the risk of trial. The plaintiff, however, is effectively less risk averse than the investors, and so bears this additional risk more lightly than they do. From the investors' perspective, the tilted variance rises with the issuance. There is more risk to spread, and spreading risk is precisely what earns the platform its fee, so it responds by attracting a larger mass of investors. On the above-parity region specified in the proposition, the proof signs this response at every candidate issuance up to the fixed-mass optimum. The plaintiff anticipates this response. His commitment benefit remains intact, while the price penalty of overselling is softened, and he therefore sells beyond $\gamma_{\mathrm{fix}}(m_{\mathrm{pl}}^{\mathrm{eq}})$.

Intermediation thus encourages overselling under the conditions of Proposition 5 and, with it, raises the share of disputes that proceed to trial. Syndication of this kind, however, need not require a formal market. Litigation funds likewise spread their risk, both across a portfolio of claims and across their own investors. The mechanism identified here may therefore already be at work in private markets for legal claims.

The effect of an endogenous platform fee on issuance remains an open question, which we are currently investigating.

6 Welfare

Throughout this section, we again restrict attention to the separating equilibrium and, for reasons of tractability, exclusively consider the representative-investor benchmark. Welfare is an important consideration even though the model involves only transfers between the agents, since the costs of bearing risk are real rather than purely distributional. Financing acts on several dimensions of the legal system at once. It shapes access to justice, changes the share of cases that proceed to court, affects the risk allocation and redistributes between the litigants. The public nature of the financing and the informativeness of the public price from the investors' due diligence act on these same dimensions. A more informative price makes risky cases more attractive to file and, at the same time, sends a larger share of them to trial, but also informs the defendant. Because financing and price informativeness act on so many dimensions at once, and because the benefits of access to justice and the costs of court trials are hard to measure, a welfare analysis considering all of the above at the same time would be neither tractable nor informative. We therefore categorize the effects, separating the risk allocation and legal activity.

Risk allocation Financing affects both the amount of risk the litigation generates and its allocation across the parties. The plaintiff oversells to strengthen his bargaining position, which lowers the good type's acceptance probability π_g and raises the share of disputes that proceed to trial. A more informative price raises this share further, but at the same time reduces the residual uncertainty about the award, so each trial carries less risk a priori. Financing also spreads the risk between the plaintiff and the investor, who bears the share γ of the claim. This spreading of risk is an improvement even though, due to overselling, the equilibrium share is not the optimal one. We ask when the public market, with its more informative price and its potentially smaller equilibrium share, allocates risk better than the private market.

More formally, we are interested in conditions under which financing improves the risk allocation. We assume that without public price revelation a less informative signal s_0 is produced with informativeness $\tau_0 \leq \tau$. One motivation for this less informative signal is that s_0 could capture information the plaintiff privately obtains from his own lawyers, rather than information revealed through public trading. We make comparisons between three cases, (i) public markets with $(\gamma^{\text{eq}}_{\text{rep}}(\tau), \tau)$, (ii) private markets with $(\gamma^{\text{eq}}_{\text{rep}}(\tau_0), \tau_0)$ and (iii) full retention with $(0, \tau_0)$. Here, we let $\gamma^{\text{eq}}_{\text{rep}}(\tau_0)$ denote the optimally chosen financing in case of private markets with $\gamma^{\text{eq}}_{\text{rep}}(\tau) < \gamma^{\text{eq}}_{\text{rep}}(\tau_0)$, assuming the condition of Proposition 4 holds along the path of informativeness levels separating the two regimes. That proposition signs the derivative at a single candidate optimum, whereas the ordering of the two shares compares the endpoints of the interval $[\tau_0, \tau]$, so the condition is needed at every optimum in between. It suffices to impose it once, at the private optimum, in the uniform form that the gap there falls below the threshold $\bar{\Delta}_a$ evaluated at every share between risk parity and that optimum and at every informativeness in $[\tau_0, \tau]$. A smaller share carries a smaller gap, so the condition is then inherited at every higher informativeness and the ordering follows. Lemma 3 in the Appendix records the argument.

We first consider the aggregate risk involved in the case. From the perspective of the plaintiff and the investor, at the point of financing, the claim Y sold carries the risk

$$\text{Var}_s(Y) = \underbrace{\psi\big(1-\pi_g\big)\frac{\theta_g^2}{\tau}}_{\text{trial-outcome risk}} + \underbrace{\psi\,\pi_g\big(1-\pi_g\big)\Big(\frac{(1-\gamma)\rho_P\,\theta_g^2}{2\tau}\Big)^2}_{\text{settlement-versus-trial risk}}$$

$$+\ \underbrace{\psi(1-\psi)\Big((\theta_b-\theta_g)\,\mu_s-\frac{(1-\gamma)\rho_P}{2\tau}\big(\theta_b^2-\pi_g\,\theta_g^2\big)\Big)^2}_{\text{defendant-type risk}}.$$

From the perspective of the defendant, by contrast, knowing her own type, the variance reduces to

$$\mathrm{Var}_s(Y \mid \theta_g) = \big(1-\pi_g\big)\frac{\theta_g^2}{\tau}\ +\ \pi_g\big(1-\pi_g\big)\Big(\frac{(1-\gamma)\rho_P\,\theta_g^2}{2\tau}\Big)^2.$$

The aggregate expected cost of risk includes the defendant's trial risk weighted by the probability ψ,

$$\mathcal{R}(\gamma,\tau) = \tfrac{1}{2}\,\rho_P(1-\gamma)^2\ \mathrm{Var}_s(Y)\ +\ \tfrac{1}{2}\,\rho_I\gamma^2\ \mathrm{Var}_s(Y)\ +\ \tfrac{1}{2}\,\rho_D\,\psi\ \mathrm{Var}_s(Y \mid \theta_g).$$

The comparison of the three financing regimes again weighs a benefit against a cost. On the sharing side the ranking is clear. The coefficient $\rho_P(1-\gamma)^2+\rho_I\gamma^2$ multiplying the common variance is minimized at risk parity and increases beyond it, so among the shares the overselling plaintiff actually chooses, a smaller sale always allocates the risk better. The effect on the variance itself, by contrast, is ambiguous. The direct effect of a larger share is to harden the separating demands and depress the good type's acceptance probability, increasing the number of risky trials and thereby raising the variance. The informativeness has offsetting effects. Its direct effect on the trial reduces the residual uncertainty about the award and so derisks every trial that occurs, while its indirect effect through the acceptance probability sends more disputes to trial and thus potentially raises the variance. The two instruments moreover interact, with Proposition 4 showing that they act as substitutes. A more informative price lowers the equilibrium share. The following proposition resolves the balance, settling the financing margin through the liquidity of the investor and the information margin through a monotone crossing in the informativeness.

Proposition 6. *Consider the representative-investor benchmark in the separating equilibrium, fix a signal s, and fix the financing shares at their equilibrium levels $\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau) < \gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau_0)$.*

(i) There exists a threshold $\tilde\rho_I > 0$ and a constant $\chi \geq 1$, depending only on primitives, such that with $\tilde\rho_D \equiv \min\{1, \chi\tilde\rho_I\}$, whenever the investor is sufficiently risk-tolerant, $\rho_I \leq \tilde\rho_I$, and the defendant is sufficiently risk-tolerant, $\rho_D \leq \tilde\rho_D$, private markets reduce aggregate risk compared to full retention,

$$\mathcal{R}(\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau_0),\tau_0)\ <\ \mathcal{R}(0,\tau_0),$$

(ii) for every $\bar\gamma < 1$, there exists a threshold $\underline{\theta}_g > 0$ such that whenever $\theta_g < \underline{\theta}_g$ and the public optimum is interior with $\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau) \leq \bar\gamma$, there is a threshold $\bar\tau_{\mathcal{R}} \in (\tau_0,\infty]$ such

that, for the share $\gamma^{\text{eq}}_{\text{rep}}(\tau)$ held fixed as the informativeness varies, public markets do not increase aggregate risk compared to private markets if and only if the public price is at most moderately informative,

$$\mathcal{R}(\gamma^{\text{eq}}_{\text{rep}}(\tau), \tau) \;\leq\; \mathcal{R}(\gamma^{\text{eq}}_{\text{rep}}(\tau_0), \tau_0) \qquad \Longleftrightarrow \qquad \tau \;\leq\; \bar{\tau}_{\mathcal{R}}.$$

The proof can be found in Appendix A.

Part (i) isolates the effect of financing in the absence of additional information. Both financing regimes carry the same informativeness τ_0, so the comparison concerns only the allocation of risk and the commitment. Financing reallocates risk between the plaintiff and the investor, but it also increases the aggregate risk, since the larger share hardens the separating demands, lowers the acceptance probability and sends more disputes to trial. Two risk aversions govern the comparison. The investor's risk aversion determines the profitability of the reallocation, since it sets the cost at which the investor absorbs the risk the plaintiff sheds. The defendant's risk aversion determines the externality of the reallocation, since the additional trials increase her exposure as well. The proposition bounds the latter in terms of the former, $\tilde{\rho}_D = \min\{1, \chi\tilde{\rho}_I\}$. Under this condition the reallocation reduces the aggregate cost of risk, and private markets improve on full retention. The proof combines three bounds. Above risk parity the sharing coefficient $\rho_P(1-\gamma)^2 + \rho_I\gamma^2$ is at most its full-sale value ρ_I. The variance of the fully retained claim is bounded below by its trial-outcome component, since the acceptance probability remains bounded away from one as $\rho_D \downarrow 0$. The variances borne at the financed share are bounded above by constants independent of ρ_I and ρ_D. The constant χ is the ratio of those two upper bounds and exceeds one, which is what lets the admissible defendant risk aversion be tied to the investor threshold rather than restricted separately. For small ρ_I and ρ_D the lower bound on the retained variance dominates both upper bounds.

Part (ii) turns to the comparison between public and private markets, where the informativeness is no longer held fixed. Passing from the private to the public regime raises the informativeness of the price from τ_0 to τ and, by Proposition 4, lowers the equilibrium share from $\gamma^{\text{eq}}_{\text{rep}}(\tau_0)$ to $\gamma^{\text{eq}}_{\text{rep}}(\tau)$, so a single instrument affects two margins at once. Two upper bounds restrict the comparison. Bounding the good type's claim, $\theta_g < \underline{\theta}_g$, suppresses the trial-outcome and settlement-versus-trial components of the variance, which carry the factor θ_g^2, and leaves the transfer gap as the governing term. Bounding the share below one, $\gamma^{\text{eq}}_{\text{rep}}(\tau) \leq \bar{\gamma}$, keeps the plaintiff's retained exposure away from zero, so that a more informative price compresses that gap strictly rather than trivially. Together the two bounds isolate the effect of Proposition 4 from the several channels through which informativeness acts. A more informative price reduces the residual uncertainty of every trial by definition. By lowering the issuance, it also improves the allocation of risk between the plaintiff and the investor and, through the softer separating demands, dampens the depressing effect of issuance on the good type's acceptance. With the upper bounds, at a fixed share a more informative price hardens the separating demand directly, sending more disputes to trial and inflating the transfer gap. This channel governs the leading term, so the cost of risk rises monotonically in the informativeness at every fixed share, and public markets improve on private ones if and only if the price is at most moderately informative, $\tau \leq \bar{\tau}_{\mathcal{R}}$. The comparison is made at the public share held fixed, so the threshold inherits its dependence on that share, and the statement does not assert monotonicity along the endogenous path $\tau \mapsto \gamma_{\text{rep}}^{\text{eq}}(\tau)$.

The two parts are established on separate regions of the parameter space, and the three-regime comparison is accordingly a pair of two-way comparisons rather than a single ordering. The lower bound on the retained variance that drives Part (i) carries the factor θ_g^2, so the investor threshold $\tilde{\rho}_I$ it delivers is of that order. Taking the good type's claim small enough for Part (ii) therefore tightens the risk tolerance Part (i) demands of the investor, and the two hold jointly only when the investor's risk aversion is small, of order θ_g^2.

Legal activity. Financing does not only reallocate a given case. It changes which cases are brought and how they end, and the two margins oppose each other. On the extensive margin, by letting the plaintiff shed risk, financing raises the value of pursuing a claim and draws in cases that would otherwise be dropped. Filing is socially valuable in its own right, since the credible prospect of trial deters unlawful conduct. On the intensive margin, financing hardens the separating demand and sends more of the good type's disputes to trial, and trial consumes the resources of the litigants and the courts. A full welfare account would net the one against the other, but the extensive margin does not admit a tractable treatment. Entry and settlement respond to the same instruments through different channels, and the threshold that governs filing is not pinned down within the model without further structure. We therefore hold the filing benefit as a qualitative virtue of financing and price only the intensive margin, asking, for a given filed case with signal realization s, how the incidence of trial depends on the mode of finance. Write $T(\gamma, \tau) = \psi\big(1 - \pi_g(\gamma, \tau)\big)$ for the probability that such a case reaches trial, the good type's dispute reaching the courtroom exactly when her offer is rejected.

Proposition 7. *Fix a filed case with signal realization s. Both private and public markets send more disputes to trial than full retention,*

$$T\big(\gamma_{\text{rep}}^{\text{eq}}(\tau_0), \tau_0\big) \;>\; T(0, \tau_0) \qquad \textit{and} \qquad T\big(\gamma_{\text{rep}}^{\text{eq}}(\tau), \tau\big) \;>\; T(0, \tau_0).$$

The proof can be found in Appendix A.

The private comparison is immediate. Private markets and full retention share the informativeness τ_0, so nothing separates them but the share, and since any positive stake depresses the good type's acceptance, the financed case reaches trial more often. The public comparison is where the two margins of financing may work against each other. Passing to a public market raises the informativeness of the price from τ_0 to τ and, by Proposition 4, feeds that more informative price back into a smaller equilibrium share, so informativeness and issuance act as substitutes even as each, on its own, hardens the separating demand and sends more disputes to trial. The direct effects both increase the incidence of trial. The substitution between them, more information bought at the price of less issuance, acts in the opposite direction. The proposition shows that the direct effects dominate. The equilibrium issuance share, though reduced by the more informative price, remains strictly above the retention level of zero, so a positive stake and a more informative price both survive the passage to public markets, and public markets too produce more trials than retention.

7 Discussion

Policy implications. Much of the current regulatory debate on litigation finance concerns transparency. Courts and legislators in the United States have moved toward mandatory disclosure of third-party funding arrangements, and similar initiatives are under discussion in the European Union.[3] These initiatives, like the critique of Beisner and Rubin [2012], rest on the presumption that more information about who finances a claim, and on what terms, improves market outcomes. Our results suggest a more cautious assessment. In the model, transparency operates through the public price. The price conveys the investors' assessment of the claim to both litigants, and its informativeness τ measures how much the market reveals. Proposition 4 shows that a more informative price reduces the plaintiff's overselling, but Proposition 6 shows, at a fixed public-market issuance, that public markets improve on private markets and on full retention only if the price is at most moderately informative. Within the model, the effect of transparency is therefore not monotone: additional disclosure does not always raise the probability of settlement. A regulator choosing between opaque private funding, disclosed private funding, and exchange trading faces an interior optimum rather than a corner solution. The result is stated at a fixed issuance and does not by itself settle the comparison along the endogenous path of issuance. This conclusion is subject to a qualification. It is derived in a model where trial, though socially costly, imposes no direct cost on the litigants. The plaintiff's hardened settlement demand costs him only the risk he retains, not any expense of the trial itself. If the parties bore the costs of trial, rejecting a settlement offer would

[3]In the United States, several federal district courts require disclosure of funding arrangements by standing order or local rule, and proposed federal legislation would extend disclosure to class actions. In the European Union, the Parliament's 2022 resolution on responsible private funding of litigation recommends an authorization and disclosure regime for commercial funders.

become costly for the plaintiff, and his incentive to oversell would weaken. How much of the interior optimum survives litigant-borne trial costs is a question our framework cannot answer. We therefore offer the results as a caution against the presumption that disclosure necessarily improves outcomes, not as an argument for deliberately limiting it. Grenadier and Grenadier [2026] reach a similarly qualified conclusion by a different route, finding that neither side's preferences over disclosure are uniform, with some plaintiffs favoring it and some defendants favoring secrecy. The same qualified reading applies to the admissibility question of Avraham and Wickelgren [2018]. Within the model, whether conveying the funder's valuation, to the court through the contract or to the defendant through the price, improves outcomes depends not on whether the valuation is conveyed but on how precisely, with the threshold $\bar{\tau}_{\mathcal{R}}$ marking the point at which greater precision becomes harmful at a fixed issuance share. The location of that threshold, and indeed its existence, would change once trial costs entered the litigants' decisions.

Throughout, the plaintiff sheds his cash-flow rights while retaining full settlement authority, and this separation drives the analysis. Overselling is profitable precisely because the bargaining posture is retained while the exposure is sold. The assumption reflects prevailing doctrine, which in most jurisdictions leaves settlement authority with the claimholder, but it also identifies the contractual margin our model holds fixed. Arrangements that move control alongside the cash flows—consent rights or veto clauses over settlement, co-decision covenants, or the outright assignment of claim and authority together where the law permits it—would close the wedge at its source. An investor who can refuse the hardened demand does not have to price it, so the commitment device disappears and the discount with it. The model accordingly predicts that claims sold with control attached should trade nearer risk parity, command higher per-share prices, and settle more often. Read this way, the champerty doctrine is inverted. Prohibitions on assigning claims are usually debated as restrictions on access to justice. In our setting the prohibition of full assignment is what forces the separation of risk-bearing from authority, which gives rise to the inefficiency. The parallel to venture capital drawn in the introduction sharpens here as well. There, too, cash-flow rights are sold ahead of control rights, and the covenants that govern exit decisions are that market's answer to the same misalignment.

Empirical predictions. The mechanism yields signed predictions, and we collect them here. First, financed cases should settle less often and reach trial more often than observably similar unfinanced cases, the pattern that Lera et al. [2023] document and attribute to risk-sharing. Second, by Proposition 4, claims priced on a public exchange should carry smaller sold shares than comparable privately funded claims, and the gap should widen with proxies for the informativeness of the investors' due diligence, such as the funder's track record or the density of precedent in the case type. Third, by Proposition 5, issue size and market depth should move together. More broadly syndicated claims are larger issues, not merely better-absorbed ones. Fourth, by Proposition 2, the per-share price peaks at risk parity rather than at the largest issues, so within the overselling region issue size and per-share price should be negatively related. Fifth, by Lemma 2, defendants facing publicly priced claims should pay higher settlements, a consequence of the hardened separating demand. What has kept such predictions beyond the data is that private funding terms are unobservable. An exchange changes this, recording shares, prices and outcomes case by case, so that the market whose effects we study also produces the evidence on which those effects can be judged.

Beyond litigation, the analysis speaks to a general property of markets attached to bargaining games. Whenever a negotiating party can sell exposure to competitive outsiders while keeping control of the negotiation, liquidity and price discovery become strategic goods. Market depth supports tougher bargaining, and information amplifies it. The empty creditor of Bolton and Oehmke [2011] and the leveraged firm of Perotti and Spier [1993], whose capital structure extracts surplus from its counterparties, are instances of the same principle. In standard settings, deeper and better-informed markets are unambiguous goods. In our case, because the asset's cash flows are themselves the object of a negotiation, deeper and better-informed markets gain an additional role and can have adverse effects.

The nearest institutional analogue is insurance. The negotiation of a payout between an insurer and its policyholder is a bargaining game over an uncertain claim, and reinsurance is the market on which the negotiating party sheds exposure while retaining settlement authority. The reinsurer occupies the investor's role, and the model carries over with a change of sign, since the party accessing the market now holds the liability rather than the asset. The match, however, is not perfect. In practice the transfer often takes the form of a loss portfolio transfer, in which the reinsurer assumes the reserves and, with them, control over the handling of the claims. Ownership and authority move together, and the reinsurer's position is complete except for a cap on the aggregate losses assumed. Reinsurance thus resembles the full assignment that champerty doctrine forbids more than the pure cash-flow sale we study. This is an avenue we leave to future research.

A Proofs

Proof of Proposition 1

Before we proceed with the proof, we provide a definition of D1 selection and show that in our setup the criterion reduces to a comparison of thresholds on the plaintiff's acceptance probabilities. Fix some equilibrium and an off-path offer x that no type makes in equilibrium, and let $D(\theta, x)$ denote the set of plaintiff acceptance probabilities under which deviating to x leaves type θ at least as well off as in equilibrium. D1 eliminates type θ from the support of the plaintiff's belief at x whenever $D(\theta, x) \subsetneq D(\theta', x)$, attributing the deviation to the type that benefits under the larger set of responses. Write V_θ for type θ's equilibrium payoff and let $\hat{\pi}_\theta(x)$ solve type θ's indifference between deviating to x and its equilibrium payoff,

$$\hat{\pi}_\theta(x)\, u_D(-x) + \big(1 - \hat{\pi}_\theta(x)\big) u_D(-\bar{x}_{D,\theta}) \;=\; V_\theta.$$

This threshold depends on the fixed equilibrium only through the equilibrium payoff V_θ. Whenever we invoke it below, it is understood relative to the equilibrium under consideration. For $x < \bar{x}_{D,\theta}$ the deviation payoff on the left strictly increases in the acceptance probability, so $D(\theta, x) = [\hat{\pi}_\theta(x), 1]$, which is empty when $\hat{\pi}_\theta(x) > 1$. For $x \geq \bar{x}_{D,\theta}$ type θ weakly prefers trial to settlement at x, so its deviation payoff is at most the trial payoff $u_D(-\bar{x}_{D,\theta})$, and $D(\theta, x)$ is empty whenever type θ strictly prefers its equilibrium payoff to trial. Hence $D(\theta_b, x) \subsetneq D(\theta_g, x)$ exactly when $D(\theta_g, x)$ is nonempty and either $\hat{\pi}_g(x) < \hat{\pi}_b(x)$ or $D(\theta_b, x)$ is empty. D1 then restricts the plaintiff's belief at x to $\hat{\psi}(x) = 1$. The symmetric comparison forces $\hat{\psi}(x) = 0$. If both gain sets are empty, no type gains from the deviation under any response and D1 places no restriction. Defining the gain sets by weak rather than strict improvement is immaterial for our arguments. Every containment invoked below remains strict under either convention.

(i) Unique separating equilibrium under D1

In any separating equilibrium, the bad type defendant's offer is $x_b = \bar{x}_{P,b}$ and the plaintiff's acceptance probability is $\pi_b = 1$. The bad type prefers settlement under this offer to trial, since $\bar{x}_{D,b} > \bar{x}_{P,b}$,[4] and the plaintiff is indifferent. If instead $x_b < \bar{x}_{P,b}$, the offer is rejected, while a deviation to $\bar{x}_{P,b} + \varepsilon$ is accepted under any belief and is profitable for small $\varepsilon > 0$, since $\bar{x}_{P,b} + \varepsilon < \bar{x}_{D,b}$. If instead $x_b > \bar{x}_{P,b}$, the offer is accepted, but the bad type can profitably lower the offer to any $x' \in (\bar{x}_{P,b}, x_b)$. Finally, if $x_b = \bar{x}_{P,b}$ but the plaintiff would instead accept with probability $\pi_b < 1$, the bad type can again profitably deviate to $\bar{x}_{P,b} + \varepsilon$.

[4] To verify this for any type θ, take the difference $\bar{x}_{D,\theta} - \bar{x}_{P,\theta} = \frac{1}{2}(\rho_D + (1-\gamma)\rho_P)\theta^2\sigma_s^2 > 0$.

In any separating equilibrium passing the D1 criterion, the good type defendant's equilibrium offer is $x_g = \bar{x}_{P,g}$ and the plaintiff's acceptance probability is $\pi_g < 1$. The good type prefers settlement with this offer to trial, since $\bar{x}_{D,g} > \bar{x}_{P,g}$, and the plaintiff is indifferent. If instead $x_g > \bar{x}_{P,b}$, the offer would be accepted, but the good type defendant would profitably deviate to the bad type's cheaper offer $\bar{x}_{P,b}$, which is accepted with certainty. If instead $x_g \in (\bar{x}_{P,g}, \bar{x}_{P,b})$, the offer is accepted, but the bad type would mimic this cheaper offer, failing separation. The same holds for $x_g = \bar{x}_{P,g}$, when the plaintiff accepts with probability $\pi_g = 1$. If instead $x_g < \bar{x}_{P,g}$, the offer is rejected, and an equilibrium can be sustained by off-path beliefs with $\hat{\psi}(x) = 0$ for all $x \in (\bar{x}_{P,g}, \bar{x}_{P,b})$ whenever $\bar{x}_{D,g} < \bar{x}_{P,b}$, i.e., the good type does not gain from deviating to the bad type's offer. However, these beliefs are eliminated by the D1 criterion. To see why, take an off-path offer $x' \in (\bar{x}_{P,g}, \bar{x}_{D,g})$, an interval that is nonempty and contained in $(\bar{x}_{P,g}, \bar{x}_{P,b})$ since $\bar{x}_{P,g} < \bar{x}_{D,g} < \bar{x}_{P,b}$. The two types enter the D1 comparison with different equilibrium payoffs. The good type's offer is rejected, so her equilibrium payoff is the trial payoff, $V_g = u_D(-\bar{x}_{D,g})$. Since $x' < \bar{x}_{D,g}$, deviating to x' leaves her at least as well off under every acceptance probability, so $\hat{\pi}_g(x') = 0$ and $D(\theta_g, x') = [0, 1]$. The bad type settles at $\bar{x}_{P,b}$, so her equilibrium payoff $V_b = u_D(-\bar{x}_{P,b})$ strictly exceeds her trial payoff $u_D(-\bar{x}_{D,b})$, since $\bar{x}_{P,b} < \bar{x}_{D,b}$. Deviating to x' therefore requires a strictly positive acceptance probability to compensate her for the risk of trial, so $\hat{\pi}_b(x') > 0$ and $D(\theta_b, x') = [\hat{\pi}_b(x'), 1] \subsetneq [0, 1] = D(\theta_g, x')$. D1 thus forces $\hat{\psi}(x') = 1$. Under this belief the plaintiff accepts the offer, since $x' > \bar{x}_{P,g}$, and the good type strictly prefers the accepted offer to trial, since $x' < \bar{x}_{D,g}$. This profitable deviation eliminates every separating equilibrium with $x_g < \bar{x}_{P,g}$.

The acceptance probability π_g is pinned down by the litigants' incentive compatibility and the D1 criterion. The bad type's incentive constraint not to mimic the good type is stated in the main text and rearranges to $\pi_g \leq \bar{\pi}_g$, with $\bar{\pi}_g$ as defined in Section 4. The good type's incentive constraint not to mimic the bad type is $u_D(-\bar{x}_{P,b}) \ \leq \ \pi_g \, u_D(-\bar{x}_{P,g}) + (1 - \pi_g) \, u_D(-\bar{x}_{D,g})$. Substituting $u_D(-x) = -e^{\rho_D x}$ and solving for π_g, this holds if and only if

$$\pi_g \ \geq \ \frac{e^{\rho_D \bar{x}_{D,g}} - e^{\rho_D \bar{x}_{P,b}}}{e^{\rho_D \bar{x}_{D,g}} - e^{\rho_D \bar{x}_{P,g}}} \ = \ \underline{\pi}_g,$$

the denominator being positive because $\bar{x}_{D,g} > \bar{x}_{P,g}$. It remains to check that $\underline{\pi}_g < \bar{\pi}_g$, so that the feasible interval is nonempty. Both thresholds are values of the map $v \mapsto \big(v - e^{\rho_D \bar{x}_{P,b}}\big) / \big(v - e^{\rho_D \bar{x}_{P,g}}\big)$, with $\underline{\pi}_g$ its value at $v = e^{\rho_D \bar{x}_{D,g}}$ and $\bar{\pi}_g$ its value at $v = e^{\rho_D \bar{x}_{D,b}}$. Its derivative $\big(e^{\rho_D \bar{x}_{P,b}} - e^{\rho_D \bar{x}_{P,g}}\big) / \big(v - e^{\rho_D \bar{x}_{P,g}}\big)^2$ is strictly positive, since $\bar{x}_{P,b} > \bar{x}_{P,g}$ by Assumption 1, so the map is strictly increasing. As $\bar{x}_{D,b} > \bar{x}_{D,g}$ gives $e^{\rho_D \bar{x}_{D,b}} > e^{\rho_D \bar{x}_{D,g}}$, it follows that $\bar{\pi}_g > \underline{\pi}_g$. Hence, any $\pi_g \in [\underline{\pi}_g, \bar{\pi}_g]$ can be sustained in a separating equilibrium.

In any separating equilibrium satisfying the D1 criterion, it must hold that $\pi_g = \bar{\pi}_g$. We show the D1 criterion eliminates all separating equilibria with $\pi_g < \bar{\pi}_g$. The equilibrium payoffs are $V_b = -e^{\rho_D \bar{x}_{P,b}}$ and $V_g = -\big(\pi_g e^{\rho_D \bar{x}_{P,g}} + (1-\pi_g) e^{\rho_D \bar{x}_{D,g}}\big)$, so solving the indifference equations for $x' < \bar{x}_{D,g}$ gives

$$\hat{\pi}_b(x') = \frac{e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D \bar{x}_{P,b}}}{e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D x'}}, \qquad \hat{\pi}_g(x') = \pi_g \frac{e^{\rho_D \bar{x}_{D,g}} - e^{\rho_D \bar{x}_{P,g}}}{e^{\rho_D \bar{x}_{D,g}} - e^{\rho_D x'}}.$$

For x' close enough to $\bar{x}_{P,g}$ we have $x' < \bar{x}_{P,b} < \bar{x}_{D,b}$, so $\hat{\pi}_b(x') \in (0,1)$ and the bad type's gain set is nonempty. As $x' \downarrow \bar{x}_{P,g}$ we have $\hat{\pi}_g(x') \to \pi_g < \bar{\pi}_g$ and $\hat{\pi}_b(x') \to \bar{\pi}_g$, so for offers near $\bar{x}_{P,g}$ the thresholds are ordered $\hat{\pi}_g(x') < \hat{\pi}_b(x')$, i.e. $D(\theta_b, x') \subsetneq D(\theta_g, x')$, and D1 places belief one on the good type. The plaintiff then accepts the deviation to the offer x' with certainty. The good type prefers to deviate, since she strictly prefers the accepted offer to her equilibrium lottery. Hence, every separating equilibrium with $\pi_g < \bar{\pi}_g$ fails D1, and we obtain uniqueness.

In the unique separating equilibrium satisfying the D1 criterion, the off-path beliefs can be taken to be $\hat{\psi}(x) = 0$ everywhere. That is, the plaintiff always believes the bad type is deviating. Consider again some deviation x'. If $x' \geq \bar{x}_{D,g}$, which occurs on part of the interval when $\bar{x}_{D,g} < \bar{x}_{P,b}$, then since $\bar{\pi}_g > 0$ the good type strictly prefers her equilibrium payoff to trial. In this case, the good type's gain set is empty. If the bad type's gain set is nonempty, then $D(\theta_g, x') \subsetneq D(\theta_b, x')$ and D1 forces belief one on the bad type; if it is empty as well, D1 places no restriction and the belief $\hat{\psi}(x') = 0$ is admissible. For $x' < \bar{x}_{D,g}$, substitute $\pi_g = \bar{\pi}_g$ into $\hat{\pi}_g(x')$, and rewrite $\hat{\pi}_b(x') < \hat{\pi}_g(x')$ as

$$\frac{e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D \bar{x}_{P,b}}}{e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D x'}} < \frac{e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D \bar{x}_{P,b}}}{e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D \bar{x}_{P,g}}} \frac{e^{\rho_D \bar{x}_{D,g}} - e^{\rho_D \bar{x}_{P,g}}}{e^{\rho_D \bar{x}_{D,g}} - e^{\rho_D x'}}.$$

Rearranging yields,

$$\big(e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D \bar{x}_{P,g}}\big)\big(e^{\rho_D \bar{x}_{D,g}} - e^{\rho_D x'}\big) < \big(e^{\rho_D \bar{x}_{D,g}} - e^{\rho_D \bar{x}_{P,g}}\big)\big(e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D x'}\big),$$

and,

$$\big(e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D \bar{x}_{D,g}}\big)\big(e^{\rho_D \bar{x}_{P,g}} - e^{\rho_D x'}\big) < 0,$$

which holds because $\bar{x}_{D,b} > \bar{x}_{D,g}$ and $x' > \bar{x}_{P,g}$, so D1 again places belief one on the bad type. The criterion therefore requires $\hat{\psi}(x) = 0$ at every off-path offer $x \in (\bar{x}_{P,g}, \bar{x}_{P,b})$. Outside this interval the plaintiff's response does not depend on his belief, leaving beliefs unrestricted.

The unique separating equilibrium under D1 exists. Under these off-path beliefs, the plaintiff's acceptance rule is sequentially rational, since he is indifferent between accepting and rejecting at both equilibrium offers. Neither defendant type gains from mimicking the other, since both incentive constraints established above hold at $\pi_g = \bar{\pi}_g$.

Every other deviation is unprofitable under the worst possible belief $\hat{\psi}(x) = 0$, since it is either rejected and leads to trial or accepted at a cost above $\bar{x}_{P,b}$. Hence, we obtain the main part of the result, and can now proceed to eliminate the other equilibrium families.

(ii) Elimination of pooling and semi-separating equilibria under D1.

In any pooling equilibrium, both types offer x_{pool}, which the plaintiff accepts with probability π_{pool} under the belief equal to the prior, $\hat{\psi}(x_{pool}) = \psi$. The plaintiff's acceptance requires $x_{pool} \geq \bar{x}_{P,\psi}$. The defendant good type requires $x_{pool} \leq \bar{x}_{D,g}$, and both types require $x_{pool} \leq \bar{x}_{P,b}$, since any transfer above $\bar{x}_{P,b}$ could be profitably undercut by an offer that is accepted under every belief. In the case of $\pi_{pool} = 1$, we get

$$x_{pool} \in \left[\bar{x}_{P,\psi},\, \min\{\bar{x}_{P,b}, \bar{x}_{D,g}\}\right].$$

Since $\bar{x}_{P,\psi} \leq \bar{x}_{P,b}$, this interval is nonempty if and only if $\bar{x}_{P,\psi} \leq \bar{x}_{D,g}$. Every such pooling equilibrium can be supported by off-path beliefs assigning probability one to the bad type everywhere. Any accepted deviation costs at least $\bar{x}_{P,b} \geq x_{pool}$. Any rejected deviation yields the trial payoff, which is at most $u_D(-\bar{x}_{D,g}) \leq u_D(-x_{pool})$ for the good type, and even worse for the bad type. Hence no deviation is profitable. In the case of $\pi_{pool} \in (0, 1)$, the plaintiff's indifference forces $x_{pool} = \bar{x}_{P,\psi}$. Existence of such equilibria requires $x_{pool} \leq \bar{x}_{D,g}$ and thus $\bar{x}_{P,\psi} \leq \bar{x}_{D,g}$. The precise set of supportable π_{pool} is immaterial, because the D1 criterion eliminates every pooling profile with $\pi_{pool} \in (0, 1]$ by the same argument as in part (i). Consider a deviation $x' \in (\bar{x}_{P,g}, x_{pool})$. This interval is nonempty, since $x_{pool} \geq \bar{x}_{P,\psi} > \bar{x}_{P,g}$, where the second inequality holds because the pooled reservation lies strictly between the type reservations by Assumption 1. Solving each type's indifference between the deviation and her equilibrium lottery gives the acceptance thresholds

$$\hat{\pi}_\theta(x') = \pi_{pool} \frac{e^{\rho_D \bar{x}_{D,\theta}} - e^{\rho_D x_{pool}}}{e^{\rho_D \bar{x}_{D,\theta}} - e^{\rho_D x'}}, \qquad \theta \in \{\theta_g, \theta_b\},$$

both lying in $[0, 1)$ since $x' < x_{pool} \leq \bar{x}_{D,g} < \bar{x}_{D,b}$, with $\hat{\pi}_g(x') = 0$ exactly when $x_{pool} = \bar{x}_{D,g}$, so both gain sets are nonempty. Apart from the common factor π_{pool}, both thresholds are values of the map $v \mapsto \big(v - e^{\rho_D x_{pool}}\big)/\big(v - e^{\rho_D x'}\big)$, which is strictly increasing because $x' < x_{pool}$, so $\bar{x}_{D,g} < \bar{x}_{D,b}$ yields $\hat{\pi}_g(x') < \hat{\pi}_b(x')$, i.e. $D(\theta_b, x') \subsetneq D(\theta_g, x')$, and D1 places belief one on the good type. The plaintiff then accepts $x' > \bar{x}_{P,g}$ with certainty, and the good type strictly gains, since the certain transfer x' lies below x_{pool} and hence below the certainty equivalent of her equilibrium lottery. A pooling offer rejected with certainty cannot be an equilibrium either. The bad type would deviate to a transfer just above $\bar{x}_{P,b}$, which is accepted under every belief and lies below $\bar{x}_{D,b}$, so the deviation improves on her trial payoff. Every pooling equilibrium therefore fails D1.

There exists no semi-separating equilibrium under D1. To see why, first observe that, in any semi-separating equilibrium under D1, each defendant type mixes over at most one common and at most one type-exclusive offer. Firstly, there cannot be more than one common offer. Suppose for contradiction that two such offers exist. Mixing requires both types to be jointly indifferent between those offers, which can only be achieved by the plaintiff's rejection with certainty of both offers, since the types differ in their trial payoffs. In this case, the bad type could profitably deviate to a transfer just above $\bar{x}_{P,b}$, which is accepted under every belief. Secondly, neither can a type mix across distinct type-exclusive offers. Any such offer carries a degenerate belief. The plaintiff generically accepts or rejects with certainty, and only randomizes at his certainty equivalent offer. Two accepted offers trivially fail indifference. If one or both offers are rejected, and indifference leaves the type with her trial payoff, there are profitable deviations. The bad type deviates to a transfer just above $\bar{x}_{P,b}$. The good type is eliminated by the D1 argument applied above to $x_g < \bar{x}_{P,g}$, provided the bad type's equilibrium payoff strictly exceeds her trial payoff, which is necessary for equilibrium. In the non-generic case, the plaintiff randomizes acceptance at his certainty equivalent offer for the given type, $\bar{x}_{P,g}$ or $\bar{x}_{P,b}$. For the bad type, again, there is a profitable deviation to a transfer just above $\bar{x}_{P,b}$. For the good type, the other offer x_{ss} in the mixing support must be accepted with certainty and lie between $\bar{x}_{P,g}$ and $\min\{\bar{x}_{D,g}, \bar{x}_{P,b}\}$, which violates the bad type's incentive constraint.

Now, denote the common offer by x_{ss}. Bayes' rule requires $\hat{\psi}(x_{ss}) \in (0,1)$. The common offer x_{ss} is accepted with positive probability, $\pi_{ss} > 0$. If instead $\pi_{ss} = 0$, then the bad type could again profitably deviate. By the plaintiff's sequential rationality for $\pi_{ss} > 0$, and Assumption 1, we get $\bar{x}_{P,g} < \bar{x}_{P,\psi_{ss}} \leq x_{ss}$; the good type's willingness to settle at the common offer further gives $x_{ss} \leq \bar{x}_{D,g}$. The respective type's equilibrium payoffs are $V_\theta = \pi_{ss}\, u_D(-x_{ss}) + (1-\pi_{ss})\, u_D(-\bar{x}_{D,\theta})$. The type-exclusive offers do not matter for this proof.

The common offer thus satisfies the two properties on which the pooling argument rested, namely $\bar{x}_{P,g} < x_{ss} \leq \bar{x}_{D,g}$ and $\pi_{ss} > 0$, and both types' equilibrium payoffs are the payoffs of the lottery at that offer. The same D1 argument therefore applies. Consider a deviation $x' \in (\bar{x}_{P,g}, x_{ss})$, an interval that is nonempty by the bound just established. Solving each type's indifference between the deviation and her equilibrium payoff gives the acceptance thresholds

$$\hat{\pi}_\theta(x') = \pi_{ss} \frac{e^{\rho_D \bar{x}_{D,\theta}} - e^{\rho_D x_{ss}}}{e^{\rho_D \bar{x}_{D,\theta}} - e^{\rho_D x'}}, \qquad \theta \in \{\theta_g, \theta_b\},$$

both lying in $[0,1)$ since $x' < x_{ss} \leq \bar{x}_{D,g} < \bar{x}_{D,b}$, so both gain sets are nonempty. Apart from the common factor π_{ss}, both thresholds are values of the map $v \mapsto \big(v - e^{\rho_D x_{ss}}\big)\big/\big(v - e^{\rho_D x'}\big)$, which is strictly increasing because $x' < x_{ss}$, so $\bar{x}_{D,g} < \bar{x}_{D,b}$ yields $\hat{\pi}_g(x') < \hat{\pi}_b(x')$.

D1 places belief one on the good type. The plaintiff then accepts $x' > \bar{x}_{P,g}$ with certainty, and the good type strictly gains, since the certain transfer x' lies below x_{ss}.

$\square$

Proof of Lemma 1

For the two separating offers differentiating directly gives

$$\partial_\gamma \bar{x}_{P,\theta} \;=\; \tfrac{1}{2}\rho_P\,\theta^2\sigma_s^2 \;>\; 0, \qquad \theta \in \{\theta_g, \theta_b\}.$$

For the two separating slopes, we find the bad type's demand rising strictly faster, as

$$\partial_\gamma \bar{x}_{P,b} - \partial_\gamma \bar{x}_{P,g} \;=\; \tfrac{1}{2}\rho_P\big(\theta_b^2 - \theta_g^2\big)\sigma_s^2 \;>\; 0.$$

For the selected separating equilibrium acceptance probability π_g, we take the derivative

$$\begin{aligned}
\partial_\gamma \pi_g &= \frac{\rho_D\left[\left(e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D \bar{x}_{P,b}}\right) e^{\rho_D \bar{x}_{P,g}} \partial_\gamma \bar{x}_{P,g} - \left(e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D \bar{x}_{P,g}}\right) e^{\rho_D \bar{x}_{P,b}} \partial_\gamma \bar{x}_{P,b}\right]}{\left(e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D \bar{x}_{P,g}}\right)^2} \\
&= \frac{\rho_D}{e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D \bar{x}_{P,g}}}\left[\pi_g e^{\rho_D \bar{x}_{P,g}} \partial_\gamma \bar{x}_{P,g} - e^{\rho_D \bar{x}_{P,b}} \partial_\gamma \bar{x}_{P,b}\right].
\end{aligned}$$

The denominator is positive, and by $\pi_g < 1$ and $\bar{x}_{P,b} > \bar{x}_{P,g}$, we also have $\pi_g e^{\rho_D \bar{x}_{P,g}} < e^{\rho_D \bar{x}_{P,b}}$. Since the bad type's demand also rises faster, $\partial_\gamma \bar{x}_{P,b} > \partial_\gamma \bar{x}_{P,g}$, the bracketed term is strictly negative and $\partial_\gamma \pi_g < 0$.

For the second derivative, we write $\partial_\gamma \log \pi_g = \eta_g - \eta_b$, where

$$\eta_\theta \equiv \frac{\rho_D \partial_\gamma \bar{x}_{P,\theta}\, e^{\rho_D \bar{x}_{P,\theta}}}{e^{\rho_D \bar{x}_{D,b}} - e^{\rho_D \bar{x}_{P,\theta}}} > 0.$$

Differentiating again, and using that the slopes $\partial_\gamma \bar{x}_{P,\theta} = \frac{1}{2}\rho_P \theta^2 \sigma_s^2$ are constant in γ, gives $\partial_\gamma \eta_\theta = \eta_\theta(\eta_\theta + \rho_D\, \partial_\gamma \bar{x}_{P,\theta})$. Consequently, the second derivative is equal to

$$\begin{aligned}
\frac{\partial_\gamma^2 \pi_g}{\pi_g} &= (\partial_\gamma \log \pi_g)^2 + \partial_\gamma^2 \log \pi_g = (\eta_g - \eta_b)^2 + \partial_\gamma \eta_g - \partial_\gamma \eta_b \\
&= (\eta_g - \eta_b)^2 + \left(\eta_g^2 - \eta_b^2\right) + \rho_D(\eta_g\, \partial_\gamma \bar{x}_{P,g} - \eta_b\, \partial_\gamma \bar{x}_{P,b}) \\
&= 2\eta_g\,(\eta_g - \eta_b) + \rho_D(\eta_g\, \partial_\gamma \bar{x}_{P,g} - \eta_b\, \partial_\gamma \bar{x}_{P,b}).
\end{aligned}$$

The slopes established above give $\partial_\gamma \bar{x}_{P,b} > \partial_\gamma \bar{x}_{P,g} > 0$, and $\partial_\gamma \pi_g < 0$ is exactly $\eta_b > \eta_g$. Both terms on the right-hand side are then negative. Hence $\partial_\gamma^2 \pi_g < 0$ and π_g is strictly concave in γ.

$\square$

Proof of Proposition 2

We evaluate U_P' at risk-parity and sign it, which tells us on which side of $\gamma_{\text{rep}}^{\text{parity}}$ the optimal financing share lies. Recall

$$U_P'(\gamma) = \tilde{\mathbb{E}}_{a_I}[Y] - \tilde{\mathbb{E}}_{a_P}[Y] + \gamma\,\tilde{\mathbb{E}}_{a_I}[\partial_\gamma Y] + (1-\gamma)\,\tilde{\mathbb{E}}_{a_P}[\partial_\gamma Y] - \frac{1}{\rho_I} h_{a_I}(\gamma) - \frac{1}{\rho_P} h_{a_P}(\gamma).$$

At the risk-parity share the exposures coincide, $a_I(\gamma_{\text{rep}}^{\text{parity}}) = a_P(\gamma_{\text{rep}}^{\text{parity}})$. It follows that the plaintiff's and the investor's terms are equal. Hence, the first two tilted Y-terms cancel out. The subsequent two terms can be combined. The last two terms are equal to zero by the indifference condition on the plaintiff accepting the offer. It follows that

$$U_P'(\gamma_{\text{rep}}^{\text{parity}}) = \tilde{\mathbb{E}}_{a_P}[\partial_\gamma Y].$$

In order to sign this expression, we write its components in terms of the primitives and the bargaining outcomes.

$$\begin{aligned}
\mathbb{E}_s\big[e^{-a_P Y}\big] &= \psi\big[\pi_g(\gamma) e^{-a_P \bar{x}_{P,g}(\gamma)} + (1-\pi_g(\gamma))\,\mathbb{E}_s\big[e^{-a_P \theta_g J}\big]\big] \;+ (1-\psi) e^{-a_P \bar{x}_{P,b}(\gamma)} \\
\mathbb{E}_s\big[\partial_\gamma Y e^{-a_P Y}\big] &= \psi \pi_g(\gamma) e^{-a_P \bar{x}_{P,g}(\gamma)} \partial_\gamma \bar{x}_{P,g}(\gamma) + (1-\psi) e^{-a_P \bar{x}_{P,b}(\gamma)} \partial_\gamma \bar{x}_{P,b}(\gamma).
\end{aligned}$$

It follows that

$$U_P'(\gamma_{\text{rep}}^{\text{parity}}) = \lambda_g(a_P)\,\partial_\gamma \bar{x}_{P,g} + \lambda_b(a_P)\,\partial_\gamma \bar{x}_{P,b},$$

where, for notational brevity, we used

$$\lambda_g(t) \equiv \frac{\psi \pi_g\, e^{-t\bar{x}_{P,g}}}{\mathbb{E}_s[e^{-tY}]}, \qquad \lambda_b(t) \equiv \frac{(1-\psi)\, e^{-t\bar{x}_{P,b}}}{\mathbb{E}_s[e^{-tY}]}.$$

This is strictly positive, since $\partial_\gamma \bar{x}_{P,g} > 0$ and $\partial_\gamma \bar{x}_{P,b} > 0$ by Lemma 1 and all other factors are positive. A marginal increase in γ away from risk parity therefore strictly raises the objective, so $\gamma_{\text{rep}}^{\text{parity}}$ is not optimal.

To conclude that $\gamma_{\text{rep}}^{\text{eq}}(\tau)$ is the unique maximizer, we show $U_P(\gamma)$ strictly concave above risk parity and also strictly increasing until risk parity. Let $\tilde{\text{Cov}}_t[.]$, $\tilde{\text{Var}}_t[.]$ denote the exponentially tilted moments with respect to risk aversion parameter t. We obtain

$$\begin{aligned}
U_P''(\gamma) &= 2\tilde{\mathbb{E}}_{a_I}[\partial_\gamma Y] + \gamma\,\tilde{\mathbb{E}}_{a_I}[\partial_\gamma^2 Y] \\
&\quad - \rho_I\Big(\tilde{\mathrm{Var}}_{a_I}(Y) + 2\gamma\,\tilde{\mathrm{Cov}}_{a_I}(Y,\partial_\gamma Y) + \gamma^2\tilde{\mathrm{Var}}_{a_I}(\partial_\gamma Y)\Big) \\
&\quad - 2\tilde{\mathbb{E}}_{a_P}[\partial_\gamma Y] + (1-\gamma)\,\tilde{\mathbb{E}}_{a_P}[\partial_\gamma^2 Y] \\
&\quad - \rho_P\Big(\tilde{\mathrm{Var}}_{a_P}(Y) - 2(1-\gamma)\,\tilde{\mathrm{Cov}}_{a_P}(Y,\partial_\gamma Y) + (1-\gamma)^2\tilde{\mathrm{Var}}_{a_P}(\partial_\gamma Y)\Big) \\
&\quad + H_{a_I}[Y] + \gamma\,H_{a_I}[\partial_\gamma Y] - H_{a_P}[Y] + (1-\gamma)\,H_{a_P}[\partial_\gamma Y] \\
&\quad - \frac{1}{\rho_I}\frac{d}{d\gamma}h_{a_I}(\gamma) - \tfrac{1}{\rho_P}\tfrac{d}{d\gamma}h_{a_P}
\end{aligned}$$

where the last two lines describe the effect of the change in the good-type acceptance probability $\pi_g(\gamma)$, with the terms

$$H_{a_j}[Z] \equiv \frac{\psi\,\partial_\gamma\pi_g(\gamma)}{\mathbb{E}_s[e^{-a_jY}]}\Big(Z_g^S\,e^{-a_j\bar{x}_{P,g}} - \mathbb{E}_s\big[Z_g^T\,e^{-a_j\theta_gJ}\big]\Big) - \tilde{\mathbb{E}}_{a_j}[Z]\,h_{a_j}(\gamma),$$

where Z_g^S and Z_g^T denote the values of Z on the good type's settlement and trial branches: $(Z_g^S, Z_g^T) = (\bar{x}_{P,g},\,\theta_gJ)$ for $Z = Y$, and $(\partial_\gamma\bar{x}_{P,g},\,0)$ for $Z = \partial_\gamma Y$. Note that these terms enter U_P'' through the change in valuation by the investor, since the plaintiff is indifferent between accepting and rejecting at each point. The term $H_{a_j}[Z]$ comes from the change in the tilted expectations with respect to the good-type acceptance probability $\pi_g(\gamma)$, where the quotient rule is applied to the tilted ratio. In order to sign U_P'', we rewrite the bracketed second-moment terms as the tilted variance of the sums of the respective random variables.

The term $-\frac{1}{\rho_P}\frac{d}{d\gamma}h_{a_P}$ vanishes because $h_{a_P} \equiv 0$ at every γ. The H_{a_P} terms cancel, since $h_{a_P} = 0$ and differentiating the plaintiff's indifference identity $e^{-a_P\bar{x}_{P,g}} = \mathbb{E}_s\big[e^{-a_P\theta_gJ}\big]$ in γ, with $a_P' = -\rho_P$, gives $\bar{x}_{P,g}\,e^{-a_P\bar{x}_{P,g}} - \mathbb{E}_s\big[\theta_gJ\,e^{-a_P\theta_gJ}\big] = (1-\gamma)\,\partial_\gamma\bar{x}_{P,g}\,e^{-a_P\bar{x}_{P,g}}$. Hence,

$$\begin{aligned}
U_P''(\gamma) &= 2\tilde{\mathbb{E}}_{a_I}[\partial_\gamma Y] - 2\tilde{\mathbb{E}}_{a_P}[\partial_\gamma Y] \\
&\quad + \gamma\,\tilde{\mathbb{E}}_{a_I}[\partial_\gamma^2 Y] + (1-\gamma)\,\tilde{\mathbb{E}}_{a_P}[\partial_\gamma^2 Y] \\
&\quad - \rho_I\,\tilde{\mathrm{Var}}_{a_I}\big(Y + \gamma\partial_\gamma Y\big) - \rho_P\,\tilde{\mathrm{Var}}_{a_P}\big(Y - (1-\gamma)\partial_\gamma Y\big) \\
&\quad + H_{a_I}[Y] + \gamma\,H_{a_I}[\partial_\gamma Y] - \frac{1}{\rho_I}\frac{d}{d\gamma}h_{a_I}(\gamma)
\end{aligned}$$

The first line is nonpositive. Integrating the identity $\partial_t\tilde{\mathbb{E}}_t[Z] = -\tilde{\mathrm{Cov}}_t(Z,Y)$ in the exposure from a_P to a_I, the first line of U_P'' becomes

$$2\big(\tilde{\mathbb{E}}_{a_I}[\partial_\gamma Y] - \tilde{\mathbb{E}}_{a_P}[\partial_\gamma Y]\big) = -2\int_{a_P}^{a_I}\tilde{\mathrm{Cov}}_t(\partial_\gamma Y,\,Y)\,dt.$$

In the overselling case the investor is the more exposed party, $a_I > a_P$, so the first line carries the opposite sign of the tilted covariance $\tilde{\mathrm{Cov}}_t(\partial_\gamma Y, Y)$ integrated over $[a_P, a_I]$. We sign this covariance, evaluated at some arbitrary tilt $t \in [a_P, a_I]$.

Define the offer gap $\Delta\bar{x} \equiv \bar{x}_{P,b} - \bar{x}_{P,g}$, the tilted branch weights $\lambda_g(t) \equiv \frac{\psi\pi_g\, e^{-t\bar{x}_{P,g}}}{\mathbb{E}_s[e^{-tY}]}$ and $\lambda_b(t) \equiv \frac{(1-\psi)e^{-t\bar{x}_{P,b}}}{\mathbb{E}_s[e^{-tY}]}$ and $\lambda_T(t) \equiv 1 - \lambda_g(t) - \lambda_b(t) \geq 0$ for the good type's trial weight, and the tilted trial mean $\tilde{\mathbb{E}}_t[\theta_g J] \equiv \mathbb{E}_s[\theta_g J e^{-t\theta_g J}]/\mathbb{E}_s[e^{-t\theta_g J}] = \theta_g\mu_s - t\,\theta_g^2\sigma_s^2$.

We rearrange as follows

$$
\begin{aligned}
\tilde{\mathrm{Cov}}_t(\partial_\gamma Y,\, Y) &= \mathbb{E}_s\Big[\partial_\gamma Y\,\big(Y - \tilde{\mathbb{E}}_t[Y]\big)e^{-tY}\Big]\,\frac{1}{\mathbb{E}_s[e^{-tY}]} \\
&= \frac{\psi\pi_g e^{-t\bar{x}_{P,g}}}{\mathbb{E}_s[e^{-tY}]}\partial_\gamma\bar{x}_{P,g}\big(\bar{x}_{P,g} - \tilde{\mathbb{E}}_t[Y]\big) + \frac{(1-\psi)e^{-t\bar{x}_{P,b}}}{\mathbb{E}_s[e^{-tY}]}\partial_\gamma\bar{x}_{P,b}\big(\bar{x}_{P,b} - \tilde{\mathbb{E}}_t[Y]\big) \\
&= \sum_{\theta\in\{\theta_g,\theta_b\}} \lambda_\theta(t)\partial_\gamma\bar{x}_{P,\theta}\big(\bar{x}_{P,\theta} - \tilde{\mathbb{E}}_t[Y]\big) \\
&= \lambda_g\partial_\gamma\bar{x}_{P,g}\big(\bar{x}_{P,g} - \lambda_g\bar{x}_{P,g} - \lambda_b\bar{x}_{P,b} - \lambda_T\tilde{\mathbb{E}}_t[\theta_g J]\big) \\
&\quad + \lambda_b\partial_\gamma\bar{x}_{P,b}\big(\bar{x}_{P,b} - \lambda_g\bar{x}_{P,g} - \lambda_b\bar{x}_{P,b} - \lambda_T\tilde{\mathbb{E}}_t[\theta_g J]\big) \\
&= \lambda_g\partial_\gamma\bar{x}_{P,g}\Big[-\lambda_b\Delta\bar{x} + \lambda_T(\bar{x}_{P,g} - \tilde{\mathbb{E}}_t[\theta_g J])\Big] \\
&\quad + \lambda_b\partial_\gamma\bar{x}_{P,b}\Big[\lambda_g\Delta\bar{x} + \lambda_T(\bar{x}_{P,b} - \tilde{\mathbb{E}}_t[\theta_g J])\Big] \\
&= \lambda_g\lambda_b\,(\bar{x}_{P,b} - \bar{x}_{P,g})\,\big(\partial_\gamma\bar{x}_{P,b} - \partial_\gamma\bar{x}_{P,g}\big) + \lambda_T \sum_{\theta\in\{\theta_g,\theta_b\}} \lambda_\theta\,\partial_\gamma\bar{x}_{P,\theta}\,\big(\bar{x}_{P,\theta} - \tilde{\mathbb{E}}_t[\theta_g J]\big),
\end{aligned}
$$

where the last two steps decompose the tilted mean over the three branches, $\tilde{\mathbb{E}}_t[Y] = \lambda_g\bar{x}_{P,g} + \lambda_b\bar{x}_{P,b} + \lambda_T\tilde{\mathbb{E}}_t[\theta_g J]$. The first term is nonnegative by Assumption 1. The second term is positive since $\bar{x}_{P,g} - \tilde{\mathbb{E}}_t[\theta_g J] = \big(t - \frac{a_P}{2}\big)\theta_g^2\sigma_s^2 > 0$ for every $t \geq a_P$ with $t > 0$; at the endpoint $t = a_P = 0$, which arises only at $\gamma = 1$, the second term vanishes and strict positivity of the covariance is carried by the first term, which is strict there by Assumption 1 and the strictly positive branch weights. By $\psi \in (0,1)$ and $\pi_g \in (0,1)$ all three branch weights are strictly positive, $\tilde{\mathrm{Cov}}_t(\partial_\gamma Y, Y) > 0$ at every tilt $t \in [a_P, a_I]$ and for $\gamma > \gamma_{\mathrm{rep}}^{\mathrm{parity}}$. Thus, the first line is strictly negative.

The second line is zero because each settlement offer is affine in γ. Differentiating $\bar{x}_{P,\theta}(\gamma)$ twice in γ gives

$$
\partial_\gamma\bar{x}_{P,\theta} = \tfrac{1}{2}\rho_P\theta^2\sigma_s^2, \qquad \partial_\gamma^2\bar{x}_{P,\theta} = 0.
$$

Hence $\partial_\gamma^2 Y \equiv 0$, so both terms $\gamma\,\tilde{\mathbb{E}}_{a_I}[\partial_\gamma^2 Y] + (1-\gamma)\,\tilde{\mathbb{E}}_{a_P}[\partial_\gamma^2 Y]$ equal zero.

The third line is obviously strictly negative, by the definition of a variance and the nondegenerate joint distribution of Y and $\partial_\gamma Y$.

We show that the fourth line,

$$
H_{a_I}[Y] + \gamma\,H_{a_I}[\partial_\gamma Y] - \frac{1}{\rho_I}\frac{d}{d\gamma}h_{a_I},
$$

is nonpositive on the overselling region $a_I > a_P$, without any auxiliary curvature condition. A quick sign check does not settle it. The bracket in $H_{a_I}[Y]$ has no fixed sign, since under overselling the trial branch carries the heavier weight and the weight gap can beat the value gap, and the derivative $\frac{d}{d\gamma}h_{a_I}$ picks up further terms once the dependence of the investor's exposure $a_I = \rho_I\gamma$ on γ is retained. We argue in three steps. We first rewrite the fourth line as a sum of three terms of known sign, one negative, one positive, one nonpositive. We then show that the negative term alone outweighs the positive one, which reduces the whole question to a single scalar inequality in the acceptance probability π_g. We finally certify that inequality, the curvature of π_g being generated at a strictly steeper scale than the level term it must dominate.

Differentiating $h_{a_I} = \psi\,\partial_\gamma\pi_g\big(e^{-a_I\bar{x}_{P,g}} - \mathbb{E}_s\big[e^{-a_I\theta_g J}\big]\big)/\mathbb{E}_s\big[e^{-a_I Y}\big]$ in γ by the product and quotient rules, keeping the pieces from the moving exposure $a_I = \rho_I\gamma$ and from the tilt in the denominator, and adding $H_{a_I}[Y] + \gamma\, H_{a_I}[\partial_\gamma Y]$, a collection of terms gives

$$
\begin{aligned}
&H_{a_I}[Y] + \gamma\, H_{a_I}[\partial_\gamma Y] - \frac{1}{\rho_I}\frac{d}{d\gamma}h_{a_I} \\
&= -\frac{\psi\,\partial_\gamma^2\pi_g}{\rho_I}\,\frac{e^{-a_I\bar{x}_{P,g}} - \mathbb{E}_s\big[e^{-a_I\theta_g J}\big]}{\mathbb{E}_s[e^{-a_I Y}]} \\
&\quad + \frac{1}{\rho_I}\,h_{a_I}^2 \\
&\quad + \frac{2\psi\,\partial_\gamma\pi_g}{\mathbb{E}_s[e^{-a_I Y}]}\Big[\, e^{-a_I\bar{x}_{P,g}}\big(\bar{x}_{P,g} + \gamma\,\partial_\gamma\bar{x}_{P,g} - \tilde{\mathbb{E}}_{a_I}[Y + \gamma\,\partial_\gamma Y]\big) \\
&\qquad\qquad + \mathbb{E}_s\big[e^{-a_I\theta_g J}\big]\,\big(\tilde{\mathbb{E}}_{a_I}[Y + \gamma\,\partial_\gamma Y] - \theta_g\mu_s + a_I\theta_g^2\sigma_s^2\big)\Big].
\end{aligned}
$$

The first term is negative: $\partial_\gamma^2\pi_g < 0$ by Lemma 1 and $e^{-a_I\bar{x}_{P,g}} < \mathbb{E}_s\big[e^{-a_I\theta_g J}\big]$ under overselling, so the numerator is negative and the leading minus sign makes the term negative. The second term is a square and hence positive. The third term is nonpositive. The tilted average $\tilde{\mathbb{E}}_{a_I}[Y + \gamma\,\partial_\gamma Y]$ is a convex combination of the good type's settlement value $\bar{x}_{P,g} + \gamma\,\partial_\gamma\bar{x}_{P,g}$, the bad type's settlement value $\bar{x}_{P,b} + \gamma\,\partial_\gamma\bar{x}_{P,b}$, which is strictly larger by Assumption 1 and Lemma 1, and the good type's tilted trial value $\theta_g\mu_s - a_I\theta_g^2\sigma_s^2$, which falls short of the good type's settlement value by $\big(a_I - \frac{a_P}{2}\big)\theta_g^2\sigma_s^2 + \gamma\,\partial_\gamma\bar{x}_{P,g} > 0$. The average therefore never drops below the trial value. If the average is at most the settlement value, both summands in the bracket are nonnegative. If it exceeds the settlement value, the average exceeds the trial value by at least its excess over the settlement value. Since $\mathbb{E}_s\big[e^{-a_I\theta_g J}\big] > e^{-a_I\bar{x}_{P,g}}$, the bracket is then at least $\big(\mathbb{E}_s\big[e^{-a_I\theta_g J}\big] - e^{-a_I\bar{x}_{P,g}}\big)$ times the excess of the average over the settlement value, which is positive. Either way the bracket is nonnegative, and multiplied by $\partial_\gamma\pi_g < 0$ the third term is nonpositive.

It remains to show that the first term dominates the second. The two share the positive factor $\psi\big(\mathbb{E}_s\big[e^{-a_I\theta_g J}\big] - e^{-a_I\bar{x}_{P,g}}\big)/\big(\rho_I\,\mathbb{E}_s\big[e^{-a_I Y}\big]\big)$, so it suffices to show the following inequality

$$\psi\,(\partial_\gamma \pi_g)^2 \big(\mathbb{E}_s\big[e^{-a_I \theta_g J}\big] - e^{-a_I \bar{x}_{P,g}}\big) \;\le\; \big|\partial^2_\gamma \pi_g\big|\,\mathbb{E}_s\big[e^{-a_I Y}\big]\,.$$

The proof now rests on this inequality. Bounding $\mathbb{E}_s\big[e^{-a_I Y}\big]$ from below by the good type's two branches, $\mathbb{E}_s\big[e^{-a_I Y}\big] \ge \psi\big[\pi_g\, e^{-a_I \bar{x}_{P,g}} + (1-\pi_g)\,\mathbb{E}_s\big[e^{-a_I\theta_g J}\big]\big]$, and canceling ψ, it suffices to prove

$$(\partial_\gamma \pi_g)^2 \big(\mathbb{E}_s\big[e^{-a_I \theta_g J}\big] - e^{-a_I \bar{x}_{P,g}}\big) \;\le\; \big|\partial^2_\gamma \pi_g\big|\big[\pi_g\, e^{-a_I \bar{x}_{P,g}} + (1-\pi_g)\,\mathbb{E}_s\big[e^{-a_I\theta_g J}\big]\big].$$

Using $\big|\partial^2_\gamma \pi_g\big| = -\partial^2_\gamma \pi_g$, we rearrange to

$$\frac{\mathbb{E}_s\big[e^{-a_I \theta_g J}\big]}{e^{-a_I \bar{x}_{P,g}}} \;\le\; \frac{(\partial_\gamma \pi_g)^2 - \partial^2_\gamma \pi_g\, \pi_g}{(\partial_\gamma \pi_g)^2 + \partial^2_\gamma \pi_g\,(1-\pi_g)}$$

whenever the right-hand denominator is positive. If it is nonpositive the inequality holds outright, since the right-hand numerator is always positive.

Consider the left-hand side of the inequality. The normal moment generating function and $\bar{x}_{P,g} = \theta_g \mu_s - \frac{a_P}{2}\theta_g^2 \sigma_s^2$ give

$$\frac{\mathbb{E}_s\big[e^{-a_I \theta_g J}\big]}{e^{-a_I \bar{x}_{P,g}}} = e^{\frac{a_I}{2}(a_I - a_P)\theta_g^2\sigma_s^2} \;\le\; e^{\frac{\rho_I^2}{2}\theta_g^2\sigma_s^2} \;<\; e^{\frac{\rho_D^2}{2}\theta_b^2\sigma_s^2} \;\le\; e^{\frac{\rho_D}{2}(\rho_D + a_P)\theta_b^2\sigma_s^2} \;<\; e^{\rho_D(\bar{x}_{D,b} - \bar{x}_{P,g})},$$

the first step from $a_I(a_I - a_P) \le a_I^2 \le \rho_I^2$ on the overselling region, the second from $\rho_I < \rho_D$ by Assumption 2 and $\theta_g < \theta_b$, the third from $a_P \ge 0$. The fourth step holds because

$$\rho_D(\bar{x}_{D,b} - \bar{x}_{P,g}) - \tfrac{\rho_D}{2}(\rho_D + a_P)\theta_b^2\sigma_s^2 = \rho_D(\theta_b - \theta_g)\Big[\mu_s - \tfrac{a_P}{2}(\theta_b + \theta_g)\sigma_s^2\Big] > 0,$$

the bracket being positive by Assumption 1 and $a_P = (1-\gamma)\rho_P \le \rho_P$.

Consider the right-hand side of the inequality. For ease of notation, define $z_\theta \equiv e^{-\rho_D(\bar{x}_{D,b} - \bar{x}_{P,\theta})}$ for $\theta \in \{\theta_g, \theta_b\}$. This allows us to rewrite $\pi_g = (1-z_b)/(1-z_g)$ and the upper bound just established as $1/z_g$. We proceed by showing the inequality. Rearranging and signing yields

$$
\begin{aligned}
&z_g\Big[(\partial_\gamma\pi_g)^2-\partial_\gamma^2\pi_g\,\pi_g\Big]-\Big[(\partial_\gamma\pi_g)^2+\partial_\gamma^2\pi_g(1-\pi_g)\Big]\\
&\quad=-(1-z_g)\pi_g^2(\eta_b-\eta_g)^2-\big[z_g\pi_g+(1-\pi_g)\big]\partial_\gamma^2\pi_g\\
&\quad=-\pi_g(1-z_b)(\eta_b-\eta_g)^2+\pi_g z_b\Big[2\eta_g(\eta_b-\eta_g)+\rho_D\big(\eta_b\partial_\gamma\bar{x}_{P,b}-\eta_g\partial_\gamma\bar{x}_{P,g}\big)\Big]\\
&\quad=\pi_g\eta_g\Big[2\eta_b-(1+z_b)\eta_g-z_b\rho_D\partial_\gamma\bar{x}_{P,g}\Big]\\
&\quad=\frac{\pi_g z_g\,\rho_D\partial_\gamma\bar{x}_{P,g}}{(1-z_b)(1-z_g)^2}\Big[2z_b\rho_D\partial_\gamma\bar{x}_{P,b}(1-z_g)-\rho_D\partial_\gamma\bar{x}_{P,g}(1-z_b)(z_b+z_g)\Big]\\
&\quad=\frac{\pi_g z_g\,\rho_D\partial_\gamma\bar{x}_{P,g}}{(1-z_b)(1-z_g)^2}\Big[2z_b\rho_D\big(\partial_\gamma\bar{x}_{P,b}-\partial_\gamma\bar{x}_{P,g}\big)(1-z_g)\\
&\qquad\qquad\qquad+\rho_D\partial_\gamma\bar{x}_{P,g}(z_b-z_g)(1+z_b)\Big]>0.
\end{aligned}
$$

The five steps of the display use the identity $\pi_g(1-z_g)=1-z_b$ from the definition of π_g, the definition of η_θ with numerator and denominator divided by $e^{\rho_D\bar{x}_{D,b}}$, and the derivatives

$$
\eta_\theta=\rho_D\,\partial_\gamma\bar{x}_{P,\theta}\,\frac{z_\theta}{1-z_\theta}\quad\text{for }\theta\in\{\theta_g,\theta_b\},\qquad \partial_\gamma\pi_g=-\pi_g(\eta_b-\eta_g),
$$

$$
\partial_\gamma^2\pi_g=-\pi_g\Big(2\eta_g(\eta_b-\eta_g)+\rho_D\big(\eta_b\partial_\gamma\bar{x}_{P,b}-\eta_g\partial_\gamma\bar{x}_{P,g}\big)\Big)<0.
$$

The first step substitutes only the first derivative and collects the two remaining terms. The second step evaluates these two factors and inserts the second derivative. The third step uses $z_b\rho_D\partial_\gamma\bar{x}_{P,b}=(1-z_b)\eta_b$, which is the definition of η_b rearranged, in the last product. The two η_b^2 terms then cancel against each other. The fourth step writes the η terms in terms of the z terms, so that the leading η_g becomes the factor $z_g\rho_D\partial_\gamma\bar{x}_{P,g}/(1-z_g)$ in front, and clearing the remaining denominator $(1-z_b)(1-z_g)$ inside the bracket gives the equation. The fifth step adds and subtracts $2z_b\rho_D\partial_\gamma\bar{x}_{P,g}(1-z_g)$, which isolates the slope difference and leaves $(z_b-z_g)(1+z_b)$ as the second factor. All of its factors are positive by Assumption 1 and Lemma 1. The slopes satisfy $\partial_\gamma\bar{x}_{P,b}>\partial_\gamma\bar{x}_{P,g}>0$, and $z_b>z_g$. The fourth line is therefore strictly negative.

Together with the first three lines, $U_P''<0$ throughout $\big(\gamma_{\text{rep}}^{\text{parity}},1\big)$, so U_P is strictly concave there and the maximizer is unique.

Finally, we show that U_P is increasing in γ for $\gamma<\gamma_{\text{rep}}^{\text{parity}}$. The plaintiff's own term vanishes for every γ, $h_{a_P}\equiv 0$. Pricing the good-type trial at his own exposure a_P makes the certainty equivalent $\bar{x}_{P,g}$ worth exactly the trial itself, so the gap h_{a_P} measures is identically zero. The three remaining terms are each strictly positive for $\gamma\in(0,\gamma_{\text{rep}}^{\text{parity}})$. At $\gamma=0$ the third term vanishes with the tilt $a_I=0$, while the first two remain strictly positive, so the conclusion below is unaffected. First, the tilted mean of the proceeds falls as the exposure that sets the tilt rises, its derivative is minus the tilted variance,

$\partial_t \tilde{\mathbb{E}}_t[Y] = -\tilde{\mathrm{Var}}_t(Y) < 0$, so the less-exposed investor ($a_I < a_P$) discounts the proceeds less heavily, $\tilde{\mathbb{E}}_{a_I}[Y] > \tilde{\mathbb{E}}_{a_P}[Y]$. Second, a larger share can only raise the proceeds. Every settlement demand increases in γ at a strictly positive rate (Lemma 1) while the good-type trial payoff is unchanged, so the proceeds rise branch by branch and hence so does their average under either tilt. Third, the same under-exposure signs the investor's term. $\bar{x}_{P,g}$ is set at the plaintiff's high exposure, but at the investor's lower exposure the good-type trial is worth strictly more than that fixed amount, so the gap inside h_{a_I} is positive. Since selling more lowers the good type's willingness to accept ($\partial_\gamma \pi_g(\gamma) < 0$, Lemma 1), h_{a_I} itself is negative and, entering the plaintiff's marginal value with a minus sign, contributes positively, $-\frac{1}{\rho_I} h_{a_I} > 0$. Thus $U_P'(\gamma) > 0$ for every $\gamma \in [0, \gamma_{\text{rep}}^{\text{parity}})$, and since the derivative is still positive at risk parity, the objective is strictly increasing up to that point and no $\gamma \leq \gamma_{\text{rep}}^{\text{parity}}$ can be optimal.

□

Proof of Proposition 3

Recall that

$$U_P'(\gamma) = \tilde{\mathbb{E}}_{a_I}[Y] - \tilde{\mathbb{E}}_{a_P}[Y] + \gamma\, \tilde{\mathbb{E}}_{a_I}[\partial_\gamma Y] + (1-\gamma)\, \tilde{\mathbb{E}}_{a_P}[\partial_\gamma Y] - \frac{1}{\rho_I} h_{a_I}(\gamma) - \frac{1}{\rho_P} h_{a_P}(\gamma).$$

Considering the convergence of $\gamma \to 1$ for each term individually, we find the following, using the fact that as $a_I \to \rho_I,\ a_P \to 0$,

$$\begin{aligned}
\tilde{\mathbb{E}}_{a_I}[Y] - \tilde{\mathbb{E}}_{a_P}[Y] &\longrightarrow \tilde{\mathbb{E}}_{\rho_I}[Y] - \mathbb{E}_s[Y], \\
\gamma\, \tilde{\mathbb{E}}_{a_I}[\partial_\gamma Y] &\longrightarrow \tilde{\mathbb{E}}_{\rho_I}[\partial_\gamma Y], \\
(1-\gamma)\, \tilde{\mathbb{E}}_{a_P}[\partial_\gamma Y] &\longrightarrow 0, \\
h_{a_I}(\gamma) &\longrightarrow h_{a_I}(1) = \psi \partial_\gamma \pi_g(1)\, \frac{e^{-\rho_I \bar{x}_{P,g}(1)} - \mathbb{E}_s\big[e^{-\rho_I \theta_g J}\big]}{\mathbb{E}_s\big[e^{-\rho_I Y}\big]}.
\end{aligned}$$

We can therefore write

$$U_P'(1) = \underbrace{\tilde{\mathbb{E}}_{\rho_I}[Y] - \mathbb{E}_s[Y]}_{<0} + \tilde{\mathbb{E}}_{\rho_I}[\partial_\gamma Y] - \frac{1}{\rho_I}\, h_{a_I}(1),$$

and more explicitly

$$\begin{aligned}
&\frac{1}{\mathbb{E}_s\big[e^{-\rho_I Y}\big]} \\
&\times \Big[(1-\psi)\big(\bar{x}_{P,b} + \partial_\gamma \bar{x}_{P,b}\big) e^{-\rho_I \bar{x}_{P,b}} + \psi \pi_g \big(\bar{x}_{P,g} + \partial_\gamma \bar{x}_{P,g}\big) e^{-\rho_I \bar{x}_{P,g}} + \psi(1-\pi_g)\, \mathbb{E}_s\big[\theta_g J\, e^{-\rho_I \theta_g J}\big] \Big] \\
&\quad - \Big[(1-\psi)\bar{x}_{P,b} + \psi \pi_g\, \bar{x}_{P,g} + \psi(1-\pi_g)\, \mathbb{E}_s[\theta_g J] \Big] - \frac{\psi\, \partial_\gamma \pi_g(1)}{\rho_I}\, \frac{e^{-\rho_I \bar{x}_{P,g}} - \mathbb{E}_s\big[e^{-\rho_I \theta_g J}\big]}{\mathbb{E}_s\big[e^{-\rho_I Y}\big]}.
\end{aligned}$$

We observe that the sign is ambiguous, implying the possibility of corner solutions, whose existence the remainder of the proof establishes. Since the investor plays no role in the bargaining stage, none of the primitives entering this expression, the offers $\bar{x}_{P,\theta}(1)$, the acceptance probability π_g and its slope $\partial_\gamma \pi_g(1)$, and hence the distribution of Y, depends on ρ_I. Holding all other primitives fixed, the corner derivative $U_P'(1)$ therefore varies only through ρ_I, and we look for a threshold on it. At the corner the plaintiff retains no exposure, $a_P = 0$, so the offers reduce to the conditional means, $\bar{x}_{P,\theta}(1) = \theta\mu_s$. In particular the good type's offer coincides with the expected trial award, $\bar{x}_{P,g} = \mathbb{E}_s[\theta_g J]$, the offer gap $\bar{x}_{P,b} - \bar{x}_{P,g} = (\theta_b - \theta_g)\mu_s$ is strictly positive, the offer ordering $\bar{x}_{P,b} > \bar{x}_{P,g}$ at $\gamma = 1$ requires only $\mu_s > 0$, which Assumption 1 implies, and the slopes remain $\partial_\gamma \bar{x}_{P,\theta} = \frac{1}{2}\rho_P \theta^2 \sigma_s^2$. Since the trial award is conditionally normal, $\mathbb{E}_s\big[e^{-\rho_I \theta_g J}\big] = e^{-\rho_I \theta_g \mu_s + \theta_g^2 \sigma_s^2 \rho_I^2/2}$ and $\mathbb{E}_s\big[\theta_g J\, e^{-\rho_I \theta_g J}\big] = \big(\theta_g \mu_s - \theta_g^2 \sigma_s^2 \rho_I\big) e^{-\rho_I \theta_g \mu_s + \theta_g^2 \sigma_s^2 \rho_I^2/2}$. Multiplying the explicit expression above by the positive factor $\mathbb{E}_s\big[e^{-\rho_I Y}\big]\, e^{\rho_I \theta_g \mu_s}$, which leaves the sign of $U_P'(1)$ unchanged, and collecting the three branches therefore gives

$$
\begin{aligned}
&\mathbb{E}_s\big[e^{-\rho_I Y}\big]\, e^{\rho_I \theta_g \mu_s}\, U_P'(1) \\
&\quad = (1-\psi)\Big[\psi(\theta_b - \theta_g)\mu_s + \tfrac{1}{2}\rho_P \theta_b^2 \sigma_s^2\Big] e^{-\rho_I(\theta_b - \theta_g)\mu_s} \\
&\qquad + \psi \pi_g \Big[\tfrac{1}{2}\rho_P \theta_g^2 \sigma_s^2 - (1-\psi)(\theta_b - \theta_g)\mu_s\Big] \\
&\qquad - \psi(1-\pi_g)\Big[\theta_g^2 \sigma_s^2 \rho_I + (1-\psi)(\theta_b - \theta_g)\mu_s\Big] e^{\theta_g^2 \sigma_s^2 \rho_I^2/2} \\
&\qquad + \frac{\psi\, \partial_\gamma \pi_g(1)}{\rho_I}\Big(e^{\theta_g^2 \sigma_s^2 \rho_I^2/2} - 1\Big),
\end{aligned}
$$

where the last term is nonpositive since $\partial_\gamma \pi_g(1) < 0$ by Lemma 1. This rescaled derivative is strictly decreasing in ρ_I. Differentiating term by term,

$$
\begin{aligned}
&\frac{d}{d\rho_I}\Big(\mathbb{E}_s\big[e^{-\rho_I Y}\big]\, e^{\rho_I \theta_g \mu_s}\, U_P'(1)\Big) \\
&\quad = -(1-\psi)(\theta_b - \theta_g)\mu_s \Big[\psi(\theta_b - \theta_g)\mu_s + \tfrac{1}{2}\rho_P \theta_b^2 \sigma_s^2\Big] e^{-\rho_I(\theta_b - \theta_g)\mu_s} \\
&\qquad - \psi(1-\pi_g)\, \theta_g^2 \sigma_s^2 \Big[1 + \theta_g^2 \sigma_s^2 \rho_I^2 + (1-\psi)(\theta_b - \theta_g)\mu_s\, \rho_I\Big] e^{\theta_g^2 \sigma_s^2 \rho_I^2/2} \\
&\qquad + \psi\, \partial_\gamma \pi_g(1)\, \frac{e^{\theta_g^2 \sigma_s^2 \rho_I^2/2}\big(\theta_g^2 \sigma_s^2 \rho_I^2 - 1\big) + 1}{\rho_I^2}.
\end{aligned}
$$

The first two terms are strictly negative. In the third, the fraction is nonnegative. The function $e^z(2z-1)+1$ vanishes at $z = 0$ and increases on $z \geq 0$, its derivative being $e^z(2z+1) > 0$, and the numerator is its value at $z = \theta_g^2 \sigma_s^2 \rho_I^2/2$. The sign of $\partial_\gamma \pi_g(1)$ then makes the third term nonpositive as well, and the rescaled derivative decreases throughout. This last term is precisely the one whose sign is inaccessible before the rescaling. Clearing the tilting denominator $\mathbb{E}_s\big[e^{-\rho_I Y}\big]$ first reduces the acceptance channel to the elementary one-variable bound just used. At the boundaries, the acceptance term is of order ρ_I and vanishes, so

$$\lim_{\rho_I\to 0^+} \mathbb{E}_s\big[e^{-\rho_I Y}\big]\, e^{\rho_I\theta_g\mu_s}\, U_P'(1) = (1-\psi)\,\tfrac{1}{2}\rho_P\theta_b^2\sigma_s^2 + \psi\pi_g\,\tfrac{1}{2}\rho_P\theta_g^2\sigma_s^2 = \mathbb{E}_s[\partial_\gamma Y] > 0,$$

where the μ_s-terms cancel across the three branches since $\psi = \psi\pi_g + \psi(1-\pi_g)$: a risk-neutral investor absorbs the whole claim at no discount, and only the first-order bargaining gain remains. As $\rho_I \to \infty$ the Gaussian trial branch dominates and the rescaled derivative diverges to $-\infty$, as the tilted valuation of the unbounded trial award collapses. A strictly decreasing function with these limits crosses zero exactly once. There is hence a unique $\underline{\rho}_I$ such that $U_P'(1) > 0$ for $\rho_I < \underline{\rho}_I$ and $U_P'(1) < 0$ for $\rho_I > \underline{\rho}_I$. Since U_P is strictly concave by the proof of Proposition 2, the corner $\gamma^{\text{eq}}_{\text{rep}}(\tau) = 1$ is optimal precisely when $\rho_I \le \underline{\rho}_I$. The crossing argument runs over all $\rho_I \in (0,\infty)$, while Assumption 2 restricts the admissible range to $\rho_I < \min\{\rho_P, \rho_D\}$. The threshold is accordingly understood as capped at that bound, and the statement delivers full sale on the admissible range below it.

□

Proof of Lemma 2

Differentiating the closed forms $\bar{x}_{P,\theta} = \theta\mu_s - \frac{1}{2}a_P\theta^2/\tau$ and $\bar{x}_{D,b} = \theta_b\mu_s + \frac{1}{2}\rho_D\theta_b^2/\tau$ gives the certainty-equivalent derivatives directly,

$$\partial_\tau \bar{x}_{P,\theta} = \frac{a_P\,\theta^2}{2\tau^2} \ge 0, \qquad \partial_\tau \bar{x}_{D,b} = -\frac{\rho_D\,\theta_b^2}{2\tau^2} \le 0, \qquad \partial_\tau\partial_\gamma \bar{x}_{P,\theta} = -\frac{\rho_P\,\theta^2}{2\tau^2} \le 0.$$

The derivative of the acceptance probability is

$$\begin{aligned}\partial_\tau\pi_g &= \frac{\rho_D}{e^{\rho_D\bar{x}_{D,b}} - e^{\rho_D\bar{x}_{P,g}}}\Big[(1-\pi_g)\,e^{\rho_D\bar{x}_{D,b}}\,\partial_\tau\bar{x}_{D,b} + \pi_g\,e^{\rho_D\bar{x}_{P,g}}\,\partial_\tau\bar{x}_{P,g} - e^{\rho_D\bar{x}_{P,b}}\,\partial_\tau\bar{x}_{P,b}\Big]\\ &= \frac{\rho_D}{2\tau^2\big(e^{\rho_D\bar{x}_{D,b}} - e^{\rho_D\bar{x}_{P,g}}\big)}\Big[-\underbrace{(1-\pi_g)\,\rho_D\,\theta_b^2\,e^{\rho_D\bar{x}_{D,b}}}_{>0} - \underbrace{a_P\,\theta_b^2\,e^{\rho_D\bar{x}_{P,b}}}_{>0} + \underbrace{\pi_g\,a_P\,\theta_g^2\,e^{\rho_D\bar{x}_{P,g}}}_{>0}\Big].\end{aligned}$$

The prefactor is positive since $\bar{x}_{D,b} > \bar{x}_{P,g}$. The bracketed term is negative. If $a_P = 0$, only the first negative term remains. If $a_P > 0$, the single positive contribution is dominated by the second term alone. Combining $\bar{x}_{P,b} > \bar{x}_{P,g}$ from Assumption 1 with the model primitives $\theta_b^2 > \theta_g^2$ and $\pi_g < 1$ gives

$$a_P\,\theta_b^2\,e^{\rho_D\bar{x}_{P,b}} > a_P\,\theta_g^2\,e^{\rho_D\bar{x}_{P,g}} > \pi_g\,a_P\,\theta_g^2\,e^{\rho_D\bar{x}_{P,g}},$$

so $\partial_\tau\pi_g < 0$: a more informative posterior lowers the good type's acceptance probability.

It remains to establish the curvature of the acceptance probability. Dividing the numerator and the denominator of π_g by $e^{\rho_D\bar{x}_{D,b}}$ writes the acceptance probability in terms of the defendant's scaled settlement surpluses against the two separating demands,

$$\pi_g = \frac{1-e^{-\omega_b}}{1-e^{-\omega_g}}, \qquad \omega_\theta \equiv \rho_D\big(\bar{x}_{D,b} - \bar{x}_{P,\theta}\big), \qquad \theta \in \{\theta_g, \theta_b\}.$$

The closed forms above give

$$\omega_b = \tfrac{1}{2}\rho_D(\rho_D + a_P)\theta_b^2/\tau = 2\bar{\tau}_\pi/\tau, \qquad \omega_g = \rho_D(\theta_b - \theta_g)\mu_s + \tfrac{1}{2}\rho_D\big(\rho_D\theta_b^2 + a_P\theta_g^2\big)/\tau,$$

so each surplus is a nonnegative constant plus a multiple of $1/\tau$, whence $\partial_\tau^2\omega_\theta = -(2/\tau)\,\partial_\tau\omega_\theta$. We record three order facts. First, $\omega_g - \omega_b = \rho_D(\bar{x}_{P,b} - \bar{x}_{P,g}) > 0$ by Assumption 1. Second, comparing the coefficients on $1/\tau$, $0 < -\tau\,\partial_\tau\omega_g \le -\tau\,\partial_\tau\omega_b = \omega_b$, since $\rho_D\theta_b^2 + a_P\theta_g^2 \le (\rho_D + a_P)\theta_b^2$. Third, $\omega_b \ge 2$ holds exactly on the claimed range $\tau \le \bar{\tau}_\pi$. The hyperbolic structure pins down the curvature of each exponential term,

$$\partial_\tau^2\big(1-e^{-\omega_\theta}\big) = e^{-\omega_\theta}\Big(\partial_\tau^2\omega_\theta - \big(\partial_\tau\omega_\theta\big)^2\Big) = -\frac{e^{-\omega_\theta}}{\tau^2}\,\big(-\tau\,\partial_\tau\omega_\theta\big)\big(-\tau\,\partial_\tau\omega_\theta - 2\big).$$

Differentiating the ratio twice and collecting terms around the first derivative gives

$$\partial_\tau^2\pi_g = \frac{\partial_\tau^2\big(1-e^{-\omega_b}\big)\big(1-e^{-\omega_g}\big) - \big(1-e^{-\omega_b}\big)\,\partial_\tau^2\big(1-e^{-\omega_g}\big)}{\big(1-e^{-\omega_g}\big)^2} - \frac{2\,e^{-\omega_g}\,\partial_\tau\omega_g}{1-e^{-\omega_g}}\,\partial_\tau\pi_g.$$

The last term is strictly negative: it is -2 times the product of two strictly negative factors, $e^{-\omega_g}\partial_\tau\omega_g/(1-e^{-\omega_g}) < 0$ and the derivative $\partial_\tau\pi_g < 0$ established above. For the first term, substituting the curvature formula and multiplying through by $\tau^2 e^{\omega_b+\omega_g} > 0$ shows its numerator to be nonpositive if and only if

$$\big(-\tau\,\partial_\tau\omega_g\big)\big(-\tau\,\partial_\tau\omega_g - 2\big)\big(e^{\omega_b} - 1\big) \;\le\; \omega_b\big(\omega_b - 2\big)\big(e^{\omega_g} - 1\big).$$

On $\tau \le \bar{\tau}_\pi$ this holds. The right-hand side is nonnegative by the third order fact. If $-\tau\,\partial_\tau\omega_g \le 2$ the left-hand side is nonpositive and the inequality is immediate. Otherwise the map $x \mapsto x(x-2)$ is increasing beyond $x = 2$, so the second order fact bounds the first factor by $\omega_b(\omega_b - 2)$, while the first order fact bounds the last, $e^{\omega_b} - 1 < e^{\omega_g} - 1$. The first term is therefore nonpositive and the second strictly negative, so $\partial_\tau^2\pi_g < 0$ whenever $\tau \le \bar{\tau}_\pi$, as claimed.

□

Proof of Proposition 4

Fix the signal realization s throughout, so that μ_s is held fixed. The price informativeness acts only through the residual noise $\sigma_s^2 = 1/\tau$. Restrict attention to interior solutions of

$\gamma_{\text{rep}}^{\text{eq}}(\tau)$. By the implicit function theorem and concavity at the interior solution, we get

$$\frac{\partial \gamma_{\text{rep}}^{\text{eq}}(\tau)}{\partial \tau} = -\frac{\partial_\tau \partial_\gamma U_P(\gamma_{\text{rep}}^{\text{eq}}(\tau))}{U_P''(\gamma_{\text{rep}}^{\text{eq}}(\tau))}, \quad \text{and} \qquad U_P''(\gamma_{\text{rep}}^{\text{eq}}(\tau)) < 0.$$

Hence, by the definition of $\tau = 1/\sigma_s^2$, the sign of our comparative static is the opposite of the sign of the cross-partial $\partial_{\sigma_s}\partial_\gamma U_P(\gamma_{\text{rep}}^{\text{eq}}(\tau))$. Thus, the proposition holds if and only if $\partial_{\sigma_s}\partial_\gamma U_P(\gamma_{\text{rep}}^{\text{eq}}(\tau)) > 0$.

We therefore compute the cross-partial of the plaintiff's objective in the share and the residual noise σ_s. Recall the marginal value of issuance $\partial_\gamma U_P(\gamma)$, which, using the previously established $\partial_t \tilde{\mathbb{E}}_t[Y] = -\tilde{\text{Var}}_t(Y)$ and $h_{a_P}(\gamma) = 0$, reads

$$\partial_\gamma U_P(\gamma) = -\int_{a_P}^{a_I} \tilde{\text{Var}}_t\big(Y(\gamma)\big)\, dt + \gamma\, \tilde{\mathbb{E}}_{a_I}[\partial_\gamma Y] + (1-\gamma)\, \tilde{\mathbb{E}}_{a_P}[\partial_\gamma Y] - \frac{1}{\rho_I}\, h_{a_I}(\gamma).$$

Differentiating term by term, we obtain

$$\partial_{\sigma_s}\partial_\gamma U_P(\gamma) = -\int_{a_P}^{a_I} \partial_{\sigma_s}\tilde{\text{Var}}_t\big(Y(\gamma)\big)\, dt + \gamma\, \partial_{\sigma_s}\tilde{\mathbb{E}}_{a_I}[\partial_\gamma Y] + (1-\gamma)\, \partial_{\sigma_s}\tilde{\mathbb{E}}_{a_P}[\partial_\gamma Y] - \frac{1}{\rho_I}\, \partial_{\sigma_s} h_{a_I}.$$

We show that at risk parity, $a_I = a_P \equiv t$, this cross-partial has positive sign. Observe that

$$\partial_{\sigma_s}\partial_\gamma U_P\big(\gamma_{\text{rep}}^{\text{parity}}\big) = \partial_{\sigma_s}\tilde{\mathbb{E}}_t[\partial_\gamma Y].$$

Using H_t from the proof of Proposition 2, we get

$$\partial_{\sigma_s}\tilde{\mathbb{E}}_t[\partial_\gamma Y] = \tilde{\mathbb{E}}_t[\partial_{\sigma_s}\partial_\gamma Y] - t\, \tilde{\text{Cov}}_t\big(\partial_\gamma Y,\, \partial_{\sigma_s} Y\big) + G_t[\partial_\gamma Y], \qquad G_t[\partial_\gamma Y] \equiv \frac{\partial_{\sigma_s}\pi_g}{\partial_\gamma \pi_g}\, H_t[\partial_\gamma Y],$$

and proceed by signing each term.

The first term is positive. At every share γ on both settlement branches, we have $\partial_{\sigma_s}\partial_\gamma \bar{x}_{P,\theta} = \rho_P \theta^2 \sigma_s > 0$. In the trial branch, it holds that $\partial_\gamma Y = 0$.

The second term is positive. We decompose the cross-covariance as

$$\begin{aligned}
\tilde{\text{Cov}}_t\big(\partial_\gamma Y,\, \partial_{\sigma_s} Y\big) &= \frac{\mathbb{E}_s\big[\partial_\gamma Y\, \big(\partial_{\sigma_s} Y - \tilde{\mathbb{E}}_t[\partial_{\sigma_s} Y]\big)\, e^{-tY}\big]}{\mathbb{E}_s\big[e^{-tY}\big]} \\
&= \sum_{\theta \in \{\theta_g, \theta_b\}} \lambda_\theta\, \partial_\gamma \bar{x}_{P,\theta}\, \big(\partial_{\sigma_s}\bar{x}_{P,\theta} - \tilde{\mathbb{E}}_t[\partial_{\sigma_s} Y]\big) \\
&= \lambda_g \lambda_b\, \big(\partial_\gamma \bar{x}_{P,b} - \partial_\gamma \bar{x}_{P,g}\big)\big(\partial_{\sigma_s}\bar{x}_{P,b} - \partial_{\sigma_s}\bar{x}_{P,g}\big) \\
&\quad + \lambda_T \sum_{\theta \in \{\theta_g, \theta_b\}} \lambda_\theta\, \partial_\gamma \bar{x}_{P,\theta}\big(\partial_{\sigma_s}\bar{x}_{P,\theta} - \tilde{\mathbb{E}}_t[\theta_g \varepsilon]\big) \\
&= \tfrac{1}{2}\rho_P \sigma_s^2 \Big[-\, a_P\, \sigma_s\, \lambda_g \lambda_b\, (\theta_b^2 - \theta_g^2)^2 + \lambda_T \sum_{\theta \in \{\theta_g, \theta_b\}} \lambda_\theta\, \theta^2 \big(-\, a_P\, \theta^2 \sigma_s - \tilde{\mathbb{E}}_t[\theta_g \varepsilon]\big)\Big],
\end{aligned}$$

where the second equality drops the trial branch from the product since $\partial_\gamma Y = 0$ there, the third uses $\tilde{\mathbb{E}}_t[\partial_{\sigma_s} Y] = \lambda_g\, \partial_{\sigma_s} \bar{x}_{P,g} + \lambda_b\, \partial_{\sigma_s} \bar{x}_{P,b} + \lambda_T\, \tilde{\mathbb{E}}_t[\theta_g \varepsilon]$ as in the previous proposition, and the last substitutes $\partial_\gamma \bar{x}_{P,\theta} = \frac{1}{2}\rho_P \theta^2 \sigma_s^2$ and $\partial_{\sigma_s} \bar{x}_{P,\theta} = -a_P \theta^2 \sigma_s$ of Lemma 2. Here $\varepsilon \equiv (J - \mu_s)/\sigma_s$ denotes the standardized award residual, with $\tilde{\mathbb{E}}_t[\theta_g \varepsilon] = -t\, \theta_g^2 \sigma_s$ under the normal tilt on the trial branch. At the tilt $t = a_P$ the bracketed factors reduce to $-a_P(\theta^2 - \theta_g^2)\sigma_s \leq 0$, so the covariance is strictly negative and the second term positive.

The third term is positive. The prefactor $\partial_{\sigma_s}\pi_g / \partial_\gamma \pi_g$ is strictly negative, since $\partial_\gamma \pi_g < 0$ by Lemma 1 and $\partial_{\sigma_s}\pi_g > 0$ by the chain rule from $\partial_\tau \pi_g < 0$ of Lemma 2, the map $\tau = \sigma_s^{-2}$ being decreasing. The factor $H_t[\partial_\gamma Y]$ is strictly negative for every $t \geq a_P$. In the definition of H_t from the proof of Proposition 2, the branch values of $\partial_\gamma Y$ are $(\partial_\gamma \bar{x}_{P,g},\, 0)$, so

$$H_t[\partial_\gamma Y] = \frac{\psi\, \partial_\gamma \pi_g\, \partial_\gamma \bar{x}_{P,g}\, e^{-t\bar{x}_{P,g}}}{\mathbb{E}_s[e^{-tY}]} - \tilde{\mathbb{E}}_t[\partial_\gamma Y]\, h_t.$$

The first summand is strictly negative by $\partial_\gamma \pi_g < 0$. The second enters with a minus sign and is nonpositive, since $\tilde{\mathbb{E}}_t[\partial_\gamma Y] > 0$ and $h_t \geq 0$ for $t \geq a_P$, the bracket in h_t being nonpositive there and multiplied by $\partial_\gamma \pi_g < 0$. The negative prefactor and the negative factor give the positive sign of the term.

We now move away from parity. Two terms of $\partial_{\sigma_s}\partial_\gamma U_P(\gamma)$ admit no unconditional sign, the integrated tilted variance and the response of the acceptance term. Both vanish at parity, and both are bounded by the gap Δ_a times a quantity that remains bounded as the share varies.

Consider the first. The integral is taken over $[a_P, a_I]$, an interval whose length is the gap itself, so

$$\left| \int_{a_P}^{a_I} \partial_{\sigma_s} \tilde{\mathrm{Var}}_t\big(Y(\gamma)\big)\, dt \right| \;\leq\; \Delta_a \max_{t \in [a_P, a_I]} \left| \partial_{\sigma_s} \tilde{\mathrm{Var}}_t\big(Y(\gamma)\big) \right|.$$

Consider the second. The plaintiff's indifference identity $e^{-a_P \bar{x}_{P,g}} = \mathbb{E}_s\left[e^{-a_P \theta_g J}\right]$ holds at every level of residual noise, so the bracket in h_t and its derivative in σ_s both vanish at $t = a_P$, and hence $\partial_{\sigma_s} h_{a_I}$ vanishes at the parity share. Since $\gamma \mapsto \partial_{\sigma_s} h_{a_I}(\gamma)$ is continuously differentiable on $[\gamma_{\mathrm{rep}}^{\mathrm{parity}}, 1]$ and $\gamma - \gamma_{\mathrm{rep}}^{\mathrm{parity}} = \Delta_a / (\rho_P + \rho_I)$, the mean value theorem gives

$$\tfrac{1}{\rho_I} \left| \partial_{\sigma_s} h_{a_I}(\gamma) \right| \;\leq\; \frac{\Delta_a}{\rho_I(\rho_P + \rho_I)} \max_{u \in [\gamma_{\mathrm{rep}}^{\mathrm{parity}}, \gamma]} \left| \partial_u \partial_{\sigma_s} h_{a_I}(u) \right|.$$

A third term requires care away from parity. The parity sign of the cross-covariance rests on the cancellation $t\, \theta_g^2 - a_P\, \theta_g^2 = 0$ at the tilt $t = a_P$. The same argument therefore signs the covariance at the plaintiff's tilt at every share, $\tilde{\mathrm{Cov}}_{a_P}\big(\partial_\gamma Y, \partial_{\sigma_s} Y\big) < 0$, but not at the investor's tilt $a_I > a_P$. We accordingly split the covariance pair of the cross-partial as

$$\begin{aligned}
&- a_I\gamma\,\tilde{\mathrm{Cov}}_{a_I}\big(\partial_\gamma Y, \partial_{\sigma_s} Y\big) - a_P(1-\gamma)\,\tilde{\mathrm{Cov}}_{a_P}\big(\partial_\gamma Y, \partial_{\sigma_s} Y\big)\\
&\quad = -\big(a_I\gamma + a_P(1-\gamma)\big)\,\tilde{\mathrm{Cov}}_{a_P}\big(\partial_\gamma Y, \partial_{\sigma_s} Y\big)\\
&\qquad - a_I\gamma\,\Big(\tilde{\mathrm{Cov}}_{a_I}\big(\partial_\gamma Y, \partial_{\sigma_s} Y\big) - \tilde{\mathrm{Cov}}_{a_P}\big(\partial_\gamma Y, \partial_{\sigma_s} Y\big)\Big),
\end{aligned}$$

where the first term on the right is strictly positive at every share above parity. The second is bounded through the tilt. Since $t \mapsto \tilde{\mathrm{Cov}}_t\big(\partial_\gamma Y, \partial_{\sigma_s} Y\big)$ is continuously differentiable, being a composition of smooth closed forms, and the two tilts differ by the gap Δ_a, the mean value theorem together with $a_I\gamma \le \rho_I$ gives

$$a_I\gamma\,\Big|\tilde{\mathrm{Cov}}_{a_I}\big(\partial_\gamma Y, \partial_{\sigma_s} Y\big) - \tilde{\mathrm{Cov}}_{a_P}\big(\partial_\gamma Y, \partial_{\sigma_s} Y\big)\Big| \;\le\; \rho_I\,\Delta_a \max_{t\in[a_P,a_I]} \big|\partial_t \tilde{\mathrm{Cov}}_t\big(\partial_\gamma Y, \partial_{\sigma_s} Y\big)\big|.$$

Write

$$\begin{aligned}
\mathcal{A}_\gamma \;\equiv\; &\max_{t\in[a_P,a_I]} \big|\partial_{\sigma_s} \tilde{\mathrm{Var}}_t\big(Y(\gamma)\big)\big| \;+\; \frac{1}{\rho_I(\rho_P+\rho_I)} \max_{u\in[\gamma_{\mathrm{rep}}^{\mathrm{parity}},\gamma]} \big|\partial_u \partial_{\sigma_s} h_{a_I}(u)\big|\\
&+\;\rho_I \max_{t\in[a_P,a_I]} \big|\partial_t \tilde{\mathrm{Cov}}_t\big(\partial_\gamma Y, \partial_{\sigma_s} Y\big)\big|
\end{aligned}$$

for the combined weight of the three Δ_a-bounded terms, and $\mathcal{S}_\gamma$ for the sum of the remaining five,

$$\begin{aligned}
\mathcal{S}_\gamma \;\equiv\; &\gamma\,\tilde{\mathbb{E}}_{a_I}[\partial_{\sigma_s}\partial_\gamma Y] + (1-\gamma)\,\tilde{\mathbb{E}}_{a_P}[\partial_{\sigma_s}\partial_\gamma Y]\\
&-\big(a_I\gamma + a_P(1-\gamma)\big)\,\tilde{\mathrm{Cov}}_{a_P}\big(\partial_\gamma Y, \partial_{\sigma_s} Y\big)\\
&+\gamma\,G_{a_I}[\partial_\gamma Y] + (1-\gamma)\,G_{a_P}[\partial_\gamma Y].
\end{aligned}$$

Each of the five is strictly positive. The two cross-derivative terms are signed by the branchwise derivatives of Lemma 2, and the two G terms by the sign of $H_t[\partial_\gamma Y]$ established above; both arguments apply at either tilt $t \in \{a_P, a_I\}$. The covariance term is evaluated at the plaintiff's tilt a_P, at which the parity sign argument applies at every share. Collecting the three bounds and this decomposition,

$$\partial_{\sigma_s}\partial_\gamma U_P(\gamma) \;\ge\; \mathcal{S}_\gamma - \Delta_a\,\mathcal{A}_\gamma,$$

so the cross-partial is strictly positive whenever

$$\Delta_a \;<\; \bar{\Delta}_a \;\equiv\; \frac{\mathcal{S}_\gamma}{\mathcal{A}_\gamma}, \qquad \text{with } \bar{\Delta}_a \equiv +\infty \text{ when } \mathcal{A}_\gamma = 0,$$

which is the threshold of the proposition, evaluated at the candidate optimum. It restricts neither liability share, and it is not vacuous. Both $\mathcal{S}_\gamma$ and $\mathcal{A}_\gamma$ are continuous in the share, the branch values, the branch weights, the acceptance probability and h_{a_I} being smooth on $[\gamma_{\text{rep}}^{\text{parity}}, 1]$. As the share approaches parity, Δ_a tends to zero, $\mathcal{A}_\gamma$ stays bounded, and $\mathcal{S}_\gamma$ tends to the strictly positive parity value $\partial_{\sigma_s}\tilde{\mathbb{E}}_t[\partial_\gamma Y]$ signed above, so $\bar{\Delta}_a$ is bounded away from zero and the inequality holds on a nondegenerate interval of shares above parity.

At an interior optimum satisfying the bound we therefore have $\partial_{\sigma_s}\partial_\gamma U_P(\gamma_{\text{rep}}^{\text{eq}}(\tau)) > 0$, and by the implicit function theorem above, $\partial\gamma_{\text{rep}}^{\text{eq}}(\tau)/\partial\tau < 0$. Therefore, we obtain the result.

□

Lemma 3 and Proof

Proposition 4 signs the derivative of the optimal share at a single candidate optimum. The welfare comparison of Section 6 compares the endpoints of an interval of informativeness levels, which requires the condition at every optimum in between. Fix two levels of informativeness $\tau_0 \le \tau_1$ and let $\gamma(\tau')$ denote the plaintiff's optimal share at τ'. With $\mathcal{S}$ and $\mathcal{A}$ as in the proof of that proposition, write $\bar{\Delta}_a(\gamma', \tau') \equiv \mathcal{S}_{\gamma'}/\mathcal{A}_{\gamma'}$ for the threshold evaluated at the share γ' and the informativeness τ', and let

$$\bar{\Delta}_{a,\min} \;\equiv\; \min\big\{\bar{\Delta}_a(\gamma', \tau') : \;\gamma' \in [\gamma_{\text{rep}}^{\text{parity}}, \gamma(\tau_0)],\; \tau' \in [\tau_0, \tau_1]\big\}$$

denote that threshold minimized over the shares at or below the optimum of the lower informativeness and over the levels of the interval. The following lemma records the uniform form of the condition under which the comparative static integrates into such an ordering.

Lemma 3. *Suppose $\gamma(\tau')$ is interior, unique and continuous on $[\tau_0, \tau_1]$. If the gap at the lower informativeness satisfies $\Delta_a\big(\gamma(\tau_0)\big) < \bar{\Delta}_{a,\min}$, then the share is strictly decreasing on $[\tau_0, \tau_1]$, and in particular $\gamma(\tau_1) < \gamma(\tau_0)$ whenever $\tau_1 > \tau_0$.*

The minimum defining $\bar{\Delta}_{a,\min}$ is attained and strictly positive, since $\mathcal{S}$ and $\mathcal{A}$ are continuous on the compact rectangle over which it is taken, the maxima entering $\mathcal{A}$ varying continuously by the maximum theorem, their objectives being jointly continuous in the tilt, the share and the informativeness, and $\mathcal{S}$ is strictly positive on it, each of its five terms being strictly positive at every share weakly above parity by the sign arguments in the proof of Proposition 4. Let $\mathcal{T}$ collect the levels $\tau' \in [\tau_0, \tau_1]$ at which the share has nowhere below τ' exceeded its value at τ_0. The set contains τ_0 and is closed by continuity. At every $\tau' \in \mathcal{T}$ the share satisfies $\gamma(\tau') \le \gamma(\tau_0)$, so the pair $(\gamma(\tau'), \tau')$ lies in the rectangle, and since the gap $\Delta_a = (\rho_P + \rho_I)\big(\gamma - \gamma_{\text{rep}}^{\text{parity}}\big)$ increases in the share,

$$\Delta_a\big(\gamma(\tau')\big) \;\le\; \Delta_a\big(\gamma(\tau_0)\big) \;<\; \bar{\Delta}_{a,\min} \;\le\; \bar{\Delta}_a\big(\gamma(\tau'), \tau'\big).$$

The bound of Proposition 4 therefore applies at τ' and gives $\partial\gamma/\partial\tau < 0$ there, the implicit function theorem being applicable at each τ' since the optimum is interior with $U_P'' < 0$ by the strict concavity above parity established in Proposition 2, which also places $\gamma(\tau') \geq \gamma_{\text{rep}}^{\text{parity}}$, so the share is strictly decreasing on $\mathcal{T}$. Were $\tau^* \equiv \sup\mathcal{T}$ strictly below τ_1, the share would satisfy $\gamma(\tau^*) < \gamma(\tau_0)$ and, by continuity, would remain below $\gamma(\tau_0)$ on a right neighborhood of τ^*, contradicting the definition of τ^*. Hence $\mathcal{T} = [\tau_0, \tau_1]$, the share is strictly decreasing on all of it, and $\gamma(\tau_1) < \gamma(\tau_0)$ for $\tau_1 > \tau_0$. $\square$

Proof of Proposition 5

Fix the signal s and suppress it. Consider an equilibrium of the endogenous-mass case, characterized by the pair $(\gamma_{\text{pl}}^{\text{eq}}, m_{\text{pl}}^{\text{eq}})$.

We first make the fee condition of the statement precise. The equilibrium mass itself moves with the fee, so the bound may not be read at $m_{\text{pl}}^{\text{eq}}$ alone. We take it over the whole range of masses the platform can attract, $\mathcal{M}_* \equiv [\rho_I/\rho_P, \bar{m}]$, its lower end supplied by Assumption 3 and its upper end by the platform's first-order condition, since the marginal value of depth vanishes as the mass grows while the marginal cost stays bounded away from zero. Let

$$\bar{\phi} \;\equiv\; \min\Big\{1, \inf_{\substack{m'\in\mathcal{M}_*,\, \gamma\in[0,\gamma_{\text{pl}}^{\text{parity}}(m')] \\ \partial_\gamma(\gamma p)>0}} \frac{\partial_\gamma U_P(\gamma; m')}{\partial_\gamma(\gamma p)(\gamma; m')}\Big\},$$

the inner infimum set to one if its domain is empty. The numerator is the plaintiff's no-fee marginal value, which is continuous and strictly positive up to $\gamma_{\text{pl}}^{\text{parity}}(m')$ by the proof of Proposition 2 at the effective risk aversion ρ_I/m', and the denominator is bounded, so the ratio is bounded away from zero uniformly on the compact $\mathcal{M}_*$ and $\bar{\phi} > 0$, a threshold that no longer depends on the fee through the equilibrium it induces. For $\phi < \bar{\phi}$ the decomposition $\partial_\gamma U_P^\phi = \partial_\gamma U_P - \phi\, \partial_\gamma(\gamma p)$ is strictly positive on $[0, \gamma_{\text{pl}}^{\text{parity}}(m')]$ at every mass $m' \in \mathcal{M}_*$, so every fixed-mass optimum lies strictly above the parity of its market. It can be verified with the derivations of the second-order derivatives in the proof of Proposition 2 that concavity persists for any fee, so $\gamma_{\text{fix}}(m) > \gamma_{\text{pl}}^{\text{parity}}(m)$ is the unique maximizer of a strictly concave objective.

We proceed to the proof of the result. If $\gamma_{\text{pl}}^{\text{eq}} = 1$ the claim is immediate, since the fixed-mass optimum is interior, $\gamma_{\text{fix}}(m_{\text{pl}}^{\text{eq}}) < 1$. Otherwise the equilibrium issuance satisfies the plaintiff's first-order condition along the platform's best response,

$$\underbrace{\partial_\gamma U_P^\phi(\gamma_{\text{pl}}^{\text{eq}}; m_{\text{pl}}^{\text{eq}})}_{\text{larger issue}} + \underbrace{(1-\phi)\,\partial_m U_P(\gamma_{\text{pl}}^{\text{eq}}; m_{\text{pl}}^{\text{eq}})}_{\text{deeper market}} \;\; \underbrace{m'(\gamma_{\text{pl}}^{\text{eq}})}_{\text{market's response}} \;=\; 0.$$

Consider next the fixed-mass benchmark at the equilibrium mass $m_{\text{pl}}^{\text{eq}}$. Its first-order condition is the same one, dropping the product term for the response of the mass,

$$\partial_\gamma U_P^\phi\big(\gamma_{\text{fix}}(m_{\text{pl}}^{\text{eq}}); m_{\text{pl}}^{\text{eq}}\big) \;=\; 0.$$

Recall the fee condition of the statement, which places the fixed-mass optimum weakly above the parity of its market, $\gamma_{\text{fix}}(m_{\text{pl}}^{\text{eq}}) \geq \gamma_{\text{pl}}^{\text{parity}}(m_{\text{pl}}^{\text{eq}})$, and which is read directly at the equilibrium mass. It can be verified adapting the derivations of the second-order derivatives in the proof of Proposition 2 that the plaintiff's fixed-mass problem is strictly concave at any mass in the overselling range. Notice that in the equilibrium first-order condition, if the product of the deeper-market and market's-response terms is positive, the larger-issue term must be negative. By the strict concavity of the fixed-mass problem, therefore, the equilibrium issuance is strictly larger than the fixed-mass optimum if and only if that product is positive.

The remainder of the proof shows that the product is positive. Suppose instead that it were not. By the first-order condition the larger-issue term is then nonnegative, $\partial_\gamma U_P^\phi(\gamma_{\text{pl}}^{\text{eq}}; m_{\text{pl}}^{\text{eq}}) \geq 0$. Since the platform's incentives to spread more invert below parity, it can easily be shown that under the fee condition of the statement the equilibrium of the endogenous-mass case also exhibits overselling, $\gamma_{\text{pl}}^{\text{eq}} > \gamma_{\text{pl}}^{\text{parity}}(m_{\text{pl}}^{\text{eq}})$.

We sign the two factors of the product at the equilibrium. The first is the value of a deeper market, and it is positive at every issuance and every mass, since additional investors each carry less exposure and thereby raise the price at which the whole issue clears. The second is the slope of the platform's best response, which carries the sign of the cross derivative $\partial^2_{\gamma m} U_P$, and we sign that derivative at the equilibrium issuance and mass.

We turn to the marginal value of an investor. Only the revenue term of the platform's profit depends on the mass, through the spread exposure $a_m = \rho_I \gamma / m$ with $\partial_m a_m = -a_m/m$. Using the identity $\partial_t \tilde{\mathbb{E}}_t[Y] = -\tilde{\mathrm{Var}}_t(Y)$ from the proof of Proposition 2 and integrating it in the exposure from 0 to a_m gives

$$\partial_m U_P = \frac{\gamma}{m}\Big(p - \tilde{\mathbb{E}}_{a_m}[Y]\Big) = \frac{1}{\rho_I}\int_0^{a_m} t\, \tilde{\mathrm{Var}}_t\big(Y(\gamma)\big)\, dt \;>\; 0,$$

which is the identity claimed in the text. The retained stake is priced independently of the mass, so $\partial_m(\gamma p) = \partial_m U_P$ and the platform's marginal value of depth is ϕ times this expression. Differentiating once more in the mass,

$$\partial_m^2 U_P = -\frac{a_m^2}{\rho_I\, m}\, \tilde{\mathrm{Var}}_{a_m}\big(Y(\gamma)\big) \;<\; 0,$$

so revenue is strictly concave in the mass. With the weakly convex cost of Assumption 3 the platform's profit is strictly concave as well, and the interior best response is the unique solution of the first-order condition $\phi\,\partial_m U_P(\gamma;m) = \kappa'(m)$, the cost being twice continuously differentiable by the same assumption, so that the curvature below is defined. Next, we sign the slope of the best response. Since the profit is smooth and strictly concave at the optimum, the implicit function theorem applies to the first-order condition and delivers that the best response is differentiable, with

$$\frac{dm(\gamma)}{d\gamma} = \frac{\phi\,\partial^2_{\gamma m} U_P}{\kappa''\big(m(\gamma)\big) - \phi\,\partial^2_m U_P}.$$

The denominator is strictly positive, so the sign is that of the cross derivative. Cross-differentiating the marginal value above in the issuance, with $\partial_\gamma a_m = \rho_I/m$,

$$\partial^2_{\gamma m} U_P = \frac{a_m}{m}\,\tilde{\mathrm{Var}}_{a_m}\big(Y(\gamma)\big) + \frac{1}{\rho_I}\int_0^{a_m} t\,\partial_\gamma \tilde{\mathrm{Var}}_t\big(Y(\gamma)\big)\,dt.$$

The integral collapses to a single tilted covariance and an acceptance-probability adjustment. To see this, note that for fixed γ the tilted variance is the second derivative of the log-transform in the exposure, $\tilde{\mathrm{Var}}_t\big(Y(\gamma)\big) = \partial^2_t \ln \mathbb{E}_s\big[e^{-tY(\gamma)}\big]$. The log-transform is smooth in (t,γ), so the order of differentiation can be exchanged. Defining $g(t) \equiv \partial_\gamma \ln \mathbb{E}_s\big[e^{-tY(\gamma)}\big]$, with primes denoting derivatives in the exposure, the integrand rewrites as $t\,\partial_\gamma \tilde{\mathrm{Var}}_t\big(Y(\gamma)\big) = t\,g''(t)$. To integrate by parts, we need g and g'. Differentiating in γ gives $g(t) = -t\,\tilde{\mathbb{E}}_t[\partial_\gamma Y] + h(t)$, where $h(t)$ is the acceptance probability term evaluated at the running tilt t, and both pieces vanish at $t=0$. Differentiating in t then gives $g'(t) = -\tilde{\mathbb{E}}_t[\partial_\gamma Y] + t\,\tilde{\mathrm{Cov}}_t\big(\partial_\gamma Y, Y\big) + h'(t)$. Integrating by parts, the variance integral becomes

$$\int_0^{a_m} t\,g''(t)\,dt = a_m\,g'(a_m) - g(a_m) = a_m^2\,\tilde{\mathrm{Cov}}_{a_m}\big(\partial_\gamma Y,\, Y\big) + a_m\,h'(a_m) - h(a_m).$$

Substituting back with $a_m^2/\rho_I = \gamma\,a_m/m$, the cross derivative takes the form

$$\partial^2_{\gamma m} U_P = \frac{a_m}{m}\Big[\tilde{\mathrm{Var}}_{a_m}\big(Y(\gamma)\big) + \gamma\,\tilde{\mathrm{Cov}}_{a_m}\big(\partial_\gamma Y,\, Y\big)\Big] + \frac{1}{\rho_I}\big(a_m\,h'(a_m) - h(a_m)\big).$$

Multiplying this cross derivative by m and clearing the prefactor with $m/\rho_I = \gamma/a_m$,

$$m\,\partial^2_{\gamma m} U_P = a_m\Big[\tilde{\mathrm{Var}}_{a_m}\big(Y(\gamma)\big) + \gamma\,\tilde{\mathrm{Cov}}_{a_m}\big(\partial_\gamma Y,\, Y\big)\Big] + \gamma\Big(h'(a_m) - \frac{h(a_m)}{a_m}\Big).$$

Recall that the covariance and the acceptance term $h(t)$ were shown to be nonnegative at every tilt $t \geq a_P$ in the proof of Proposition 2. The nonnegativity of the slope $h'(t)$ on the same range is established below, where the factorization of the acceptance adjustment exhibits its three summands as positive. Hence, what remains is to show that the negative effect of subtracting h is dominated by the remaining terms.

Above the parity of the market the investors are the more exposed side, by $a_m > a_P$. Signing the level and the slope separately does not settle the adjustment, since the level enters with a minus sign. The bounds below sign the combination $a_m\, h'(a_m) - h(a_m)$.

In the following, we bound the moments bracket from below and the acceptance adjustment term in turn. To simplify notation, we write $\mu_T(t) \equiv \theta_g \mu_s - t\,\theta_g^2 \sigma_s^2$ for the tilted trial mean, and define

$$\delta(t) \equiv \tfrac{1}{2}\, t\,(t - a_P)\,\theta_g^2\sigma_s^2, \quad \text{and} \quad \delta'(t) = \big(t - \tfrac{a_P}{2}\big)\theta_g^2\sigma_s^2 = \bar{x}_{P,g} - \mu_T(t).$$

First, we bound the moments bracket from below by the trial gap, combining the branch decomposition of the covariance with the variance. The chain is

$$
\begin{aligned}
& a_m \Big[\tilde{\mathrm{Var}}_{a_m}\big(Y(\gamma)\big) + \gamma\, \tilde{\mathrm{Cov}}_{a_m}\big(\partial_\gamma Y,\, Y\big)\Big] \\
&\qquad = a_m\, \tilde{\mathrm{Var}}_{a_m}\big(Y(\gamma)\big) + \gamma a_m\, \lambda_T(a_m)\, \delta'(a_m)\, \tilde{\mathbb{E}}_{a_m}[\partial_\gamma Y] \\
&\qquad\quad + \gamma a_m\, \Delta\bar{x}\, \lambda_b(a_m) \Big(\lambda_g(a_m)\big(\partial_\gamma \bar{x}_{P,b} - \partial_\gamma \bar{x}_{P,g}\big) + \lambda_T(a_m)\, \partial_\gamma \bar{x}_{P,b}\Big) \\
&\qquad \geq a_m \lambda_b(a_m) \lambda_T(a_m) \big(\delta'(a_m) + \Delta\bar{x}\big)^2 + \gamma\, \big(a_m \lambda_T(a_m)\, \delta'(a_m)\big)\, \tilde{\mathbb{E}}_{a_m}[\partial_\gamma Y] \\
&\qquad \geq \big(a_m \lambda_T(a_m)\, \delta'(a_m)\big) \Big(\lambda_b(a_m)\, \Delta\bar{x} + \gamma\, \tilde{\mathbb{E}}_{a_m}[\partial_\gamma Y]\Big).
\end{aligned}
$$

The equality derives from the branch decomposition of the covariance in the proof of Proposition 2 at the tilt a_m, factoring out $\tilde{\mathbb{E}}_{a_m}[\partial_\gamma Y] = \lambda_b(a_m)\, \partial_\gamma \bar{x}_{P,b} + \lambda_g(a_m)\, \partial_\gamma \bar{x}_{P,g}$, the weighted sum of the offers' slopes, since $\partial_\gamma Y$ vanishes at trial. The first inequality drops the $\Delta\bar{x}$ term of the covariance, which is nonnegative since the slopes are positive, as in the proof of Proposition 2, by Assumption 1, and keeps the (bad demand, trial) pair of the pairwise form of the variance over the three branches. The second inequality absorbs the offer gap with the squared trial gap, $(\delta'(a_m) + \Delta\bar{x})^2 \geq 4\,\delta'(a_m)\Delta\bar{x} \geq \delta'(a_m)\Delta\bar{x}$, and collects the common factor $a_m \lambda_T(a_m)\, \delta'(a_m)$.

Next, we bound the potentially negative adjustment term $\gamma\big(h'(a_m) - h(a_m)/a_m\big)$ from below. Since the slope $h'(a_m)$ is nonnegative, the adjustment term is at least $-\frac{\gamma}{a_m}\, h(a_m)$, and it suffices to bound the level from above. For this purpose we use the plaintiff's marginal value gross of the fee, which we first sign by splitting it into the financing revenue and the slope of the certainty equivalent of the stake he retains,

$$(1-\phi)\, \partial_\gamma U_P = \partial_\gamma U_P^\phi - \phi\Big(\underbrace{-\tilde{\mathbb{E}}_{a_P}[Y] + (1-\gamma)\, \tilde{\mathbb{E}}_{a_P}[\partial_\gamma Y]}_{\text{retained stake}}\Big),$$

the plaintiff's own acceptance term vanishing identically by his indifference condition, as in the proof of Proposition 2. The tilted mean $\tilde{\mathbb{E}}_{a_P}[Y]$ is bounded below by its smallest branch, the tilted trial mean $\theta_g \mu_s - a_P \theta_g^2 \sigma_s^2$, which falls short of the good type's offer $\bar{x}_{P,g}$

by $\frac{a_P}{2}\theta_g^2\sigma_s^2$ and of the bad type's by more, by Assumption 1, while the offers' slopes are $\partial_\gamma \bar{x}_{P,\theta} = \frac{1}{2}\rho_P\theta^2\sigma_s^2$, so that $a_P = (1-\gamma)\rho_P \leq \rho_P/2$, from $\gamma \geq \frac{1}{2}$ by Assumption 3, gives

$$-\tilde{\mathbb{E}}_{a_P}[Y] + (1-\gamma)\,\tilde{\mathbb{E}}_{a_P}[\partial_\gamma Y] \;\leq\; -\theta_g\mu_s + \tfrac{1}{2}\rho_P\theta_g^2\sigma_s^2 + \tfrac{1}{4}\rho_P\theta_b^2\sigma_s^2 \;\leq\; 0,$$

the last inequality being the moments condition of Assumption 4. With $\partial_\gamma U_P^\phi(\gamma_{\text{pl}}^{\text{eq}}; m_{\text{pl}}^{\text{eq}}) \geq 0$ by the supposition, the marginal value gross of the fee is therefore nonnegative, and rewriting $\partial_\gamma U_P \geq 0$ makes the level comparable to the moments bound:

$$\frac{1}{\rho_I}\,h(a_m) \leq \frac{1}{m}\Big[\tilde{\mathbb{E}}_{a_m}[Y] - \tilde{\mathbb{E}}_{a_P}[Y] + \gamma\,\tilde{\mathbb{E}}_{a_m}[\partial_\gamma Y] + (1-\gamma)\,\tilde{\mathbb{E}}_{a_P}[\partial_\gamma Y]\Big].$$

The plaintiff's marginal value prices the level from above. Every term on its right-hand side has a closed form, so no estimation is needed. The chain is

$$\begin{aligned}
\frac{\gamma}{a_m}\,h(a_m) &\leq \gamma\,\tilde{\mathbb{E}}_{a_m}[\partial_\gamma Y] + (1-\gamma)\big(\lambda_b(a_P)\,\partial_\gamma\bar{x}_{P,b} + \lambda_g(a_P)\,\partial_\gamma\bar{x}_{P,g}\big)\\
&\quad + \tilde{\mathbb{E}}_{a_m}[Y] - \tilde{\mathbb{E}}_{a_P}[Y]\\
&= \gamma\,\tilde{\mathbb{E}}_{a_m}[\partial_\gamma Y] + (1-\gamma)\big(\lambda_b(a_P)\,\partial_\gamma\bar{x}_{P,b} + \lambda_g(a_P)\,\partial_\gamma\bar{x}_{P,g}\big)\\
&\quad + \big(\lambda_g(a_P) - \lambda_g(a_m)\big)\Delta\bar{x} + \lambda_T(a_P)\big(\delta'(a_P) + \Delta\bar{x}\big)\\
&\quad - \lambda_T(a_m)\big(\delta'(a_m) + \Delta\bar{x}\big)\\
&= \gamma\,\tilde{\mathbb{E}}_{a_m}[\partial_\gamma Y] + \lambda_b(a_m)\,\Delta\bar{x} - \big(\lambda_T(a_m)\,\delta'(a_m) - \delta'(a_P)\big)\\
&\quad - \lambda_b(a_P)\Big(\Delta\bar{x} - (1-\gamma)\big(\partial_\gamma\bar{x}_{P,b} - \partial_\gamma\bar{x}_{P,g}\big)\Big).
\end{aligned}$$

The first relation derives from the inequality above and the branch decomposition of the proof of Proposition 2. The second equality derives from the branch decomposition $\tilde{\mathbb{E}}_t[Y] = \lambda_b(t)\,\bar{x}_{P,b} + \lambda_g(t)\,\bar{x}_{P,g} + \lambda_T(t)\,\mu_T(t)$, and substituting the weights $\lambda_b(t) = 1 - \lambda_g(t) - \lambda_T(t)$, the offer gap $\bar{x}_{P,b} - \bar{x}_{P,g} = \Delta\bar{x}$ and the trial gap $\bar{x}_{P,b} - \mu_T(t) = \Delta\bar{x} + \delta'(t)$, rearranging and canceling out the common term $\bar{x}_{P,b}$. The third equality uses the definition of the branch weights again, plugging in for $\lambda_g(a_P)$ and rearranging. The last term is nonpositive by the assumptions from the main text. We rearrange the terms as $\Delta\bar{x} - (1-\gamma)\big(\partial_\gamma\bar{x}_{P,b} - \partial_\gamma\bar{x}_{P,g}\big) \;=\; (\theta_b - \theta_g)\big(\mu_s - a_P(\theta_b + \theta_g)\sigma_s^2\big)$. This is nonnegative exactly when $\mu_s \geq (1-\gamma)\rho_P(\theta_g + \theta_b)\sigma_s^2$. Assumption 1, evaluated at $\gamma = 0$, delivers $\mu_s > \frac{1}{2}\rho_P(\theta_g + \theta_b)\sigma_s^2$. By Assumption 3, we get $m\rho_P/(m\rho_P + \rho_I) \geq \frac{1}{2}$, and the equilibrium issuance lies strictly above it, $\gamma > \frac{1}{2}$. Dropping this nonpositive term and flipping the sign, the level bound becomes a lower bound on the adjustment term,

$$\gamma\Big(h'(a_m) - \frac{h(a_m)}{a_m}\Big) \;\geq\; -\frac{\gamma}{a_m}\,h(a_m) \;\geq\; \big(\lambda_T(a_m)\,\delta'(a_m) - \delta'(a_P)\big) - \lambda_b(a_m)\,\Delta\bar{x} - \gamma\,\tilde{\mathbb{E}}_{a_m}[\partial_\gamma Y]\,,$$

which retains the trial-gap term $\lambda_T(a_m)\,\delta'(a_m) - \delta'(a_P)$ with a positive sign.

The argument now splits into two cases on the size of the retained trial-gap term against the exposure. If it clears the reciprocal exposure, $a_m\big(\lambda_T(a_m)\,\delta'(a_m)-\delta'(a_P)\big) \geq 1$, the term is in particular positive and can be dropped from the lower bound, and the two bounds carry the same bracket. The adjustment term is at least $-\big(\lambda_b(a_m)\,\Delta\bar{x} + \gamma\,\tilde{\mathbb{E}}_{a_m}[\partial_\gamma Y]\big)$, while the moments bound carries that bracket with the factor $a_m\lambda_T(a_m)\,\delta'(a_m) = a_m\big(\lambda_T(a_m)\,\delta'(a_m) - \delta'(a_P)\big) + a_m\,\delta'(a_P) > 1$. Hence $m\,\partial^2_{\gamma m}U_P \geq \big(a_m\lambda_T(a_m)\,\delta'(a_m) - 1\big)\big(\lambda_b(a_m)\,\Delta\bar{x} + \gamma\,\tilde{\mathbb{E}}_{a_m}[\partial_\gamma Y]\big) > 0$, the bracket being strictly positive with the demands and their slopes ordered as before.

Otherwise the trial-gap term falls short of the exposure, $a_m\big(\lambda_T(a_m)\,\delta'(a_m)-\delta'(a_P)\big) < 1$, and the shortfall forces the slope of the adjustment to carry its level. We use the identity

$$e^{-t\bar{x}_{P,g}} - \mathbb{E}_s\big[e^{-t\theta_g J}\big] = -\,\mathbb{E}_s\big[e^{-t\theta_g J}\big]\,\Big(1 - e^{-\delta(t)}\Big)$$

to write $h(t)$ in factorized form. With $\lambda_T(t) = \psi(1-\pi_g)\,\mathbb{E}_s\big[e^{-t\theta_g J}\big]\,/\mathbb{E}_s\big[e^{-tY}\big]$ the tilted weight of the trial branch,

$$h(t) = r\,\lambda_T(t)\,\big(1 - e^{-\delta(t)}\big), \qquad r \equiv \frac{-\partial_\gamma \pi_g}{1-\pi_g} \;>\; 0.$$

Logarithmic differentiation splits the slope-to-level ratio into three positive summands. The tilted weight contributes $\tilde{\mathbb{E}}_t[Y] - \mu_T(t)$, decomposed over branches into the trial gaps $\delta'(t)$ and $\delta'(t) + \Delta\bar{x}$, and the exponential factor contributes its own log-derivative,

$$\frac{h'(t)}{h(t)} = \big(1 - \lambda_T(t)\big)\,\delta'(t) + \lambda_b(t)\,\Delta\bar{x} + \frac{\delta'(t)}{e^{\delta(t)} - 1} \;>\; 0.$$

Evaluate at a_m. The split $a_m\delta'(a_m) = 2\delta(a_m) + a_m\,\delta'(a_P)$ turns the first summand into $2\delta(a_m)$ minus the trial-gap term, and bounds the third summand below by $2\delta(a_m)/(e^{\delta(a_m)} - 1)$. Dropping the (nonnegative) middle summand and combining the other two via $\delta(a_m) + \delta(a_m)/(e^{\delta(a_m)} - 1) = \delta(a_m)/(1 - e^{-\delta(a_m)})$,

$$a_m\,\frac{h'(a_m)}{h(a_m)} \;\geq\; \frac{2\delta(a_m)}{1 - e^{-\delta(a_m)}} - a_m\big(\lambda_T(a_m)\,\delta'(a_m) - \delta'(a_P)\big) \;>\; 2 - 1 \;=\; 1,$$

using $1 - e^{-\delta(a_m)} < \delta(a_m)$ for $\delta(a_m) > 0$ together with the shortfall hypothesis. Hence $a_m\,h'(a_m) > h(a_m)$. The slope of the acceptance adjustment carries its level, the adjustment term is strictly positive and can be dropped, and the moments bracket alone signs the cross derivative, $m\,\partial^2_{\gamma m}U_P > a_m\,\tilde{\mathrm{Var}}_{a_m}\big(Y(\gamma)\big) > 0$.

In both cases, therefore, $\partial^2_{\gamma m}U_P > 0$ at the equilibrium issuance and mass.

Since the fee enters the implicit-function formula only through the positive factor ϕ, the platform's best response is strictly increasing at the equilibrium, $m'(\gamma^{\mathrm{eq}}_{\mathrm{pl}}) > 0$.

The deeper-market term is positive, and the market's response is strictly increasing at the equilibrium, so the product of the two is positive, contradicting the supposition. The product is therefore positive, and the equivalence of the opening delivers the claim, $\gamma_{\text{pl}}^{\text{eq}} > \gamma_{\text{fix}}(m_{\text{pl}}^{\text{eq}})$. □

Proof of Proposition 6

Throughout, the signal s is fixed, $\sigma_s^2 = 1/\tau \in (0, \sigma_0^2]$ with $\sigma_0^2 \equiv 1/\tau_0$ denotes the trial variance per unit share, $a_P(\gamma) \equiv (1-\gamma)\rho_P$ is the plaintiff's effective risk aversion, and

$$R(\gamma) \;\equiv\; \rho_P(1-\gamma)^2 + \rho_I\gamma^2$$

for the sharing coefficient, so that $\mathcal{R} = \frac{1}{2}R(\gamma)\,\mathrm{Var}_s(Y) + \frac{1}{2}\rho_D\psi\,\mathrm{Var}_s(Y \mid \theta_g)$. In the variance we abbreviate the transfer gap between the types,

$$M(\gamma, \sigma_s^2) \;\equiv\; (\theta_b - \theta_g)\,\mu_s - \tfrac{1}{2}\,a_P(\gamma)\,\sigma_s^2\,\big(\theta_b^2 - \pi_g\theta_g^2\big),$$

so that

$$\mathrm{Var}_s(Y) \;=\; \psi(1-\pi_g)\theta_g^2\sigma_s^2 + \psi\pi_g(1-\pi_g)\Big(\tfrac{1}{2}a_P(\gamma)\theta_g^2\sigma_s^2\Big)^2 + \psi(1-\psi)M^2,$$

$$\mathrm{Var}_s(Y \mid \theta_g) \;=\; (1-\pi_g)\theta_g^2\sigma_s^2 + \pi_g(1-\pi_g)\Big(\tfrac{1}{2}a_P(\gamma)\theta_g^2\sigma_s^2\Big)^2.$$

Two facts about the sharing coefficient are used repeatedly. It is the Borch parabola, minimized at risk parity with value $R(\gamma_{\text{rep}}^{\text{parity}}) = \rho_P\rho_I/(\rho_P + \rho_I) \equiv R_{\min}$, and it increases beyond parity, so that

$$R_{\min} \;\le\; R(\gamma) \;\le\; R(1) \;=\; \rho_I \qquad \text{for every } \gamma \in [\gamma_{\text{rep}}^{\text{parity}}, 1].$$

Part (i): By Proposition 2 the private share lies above parity, so $R(\gamma_{\text{rep}}^{\text{eq}}(\tau_0)) \le \rho_I$: whatever variance the financed claim carries, overselling caps the joint plaintiff–investor coefficient at the investor's risk aversion. The variances themselves are bounded uniformly over $\gamma \in [0,1]$ and $\rho_D > 0$. Using $1-\pi_g \le 1$, $\pi_g(1-\pi_g) \le \frac{1}{4}$, $a_P \le \rho_P$ and $|M| \le \bar{M} \equiv (\theta_b - \theta_g)\mu_s + \frac{1}{2}\rho_P\theta_b^2\sigma_0^2$,

$$\mathrm{Var}_s(Y) \;\le\; \bar{B} \;\equiv\; \psi\theta_g^2\sigma_0^2 + \tfrac{\psi}{4}\big(\tfrac{1}{2}\rho_P\theta_g^2\sigma_0^2\big)^2 + \psi(1-\psi)\bar{M}^2,$$

$$\mathrm{Var}_s(Y \mid \theta_g) \;\le\; \bar{B}_g \;\equiv\; \theta_g^2\sigma_0^2 + \tfrac{1}{4}\big(\tfrac{1}{2}\rho_P\theta_g^2\sigma_0^2\big)^2.$$

For the premium the plaintiff sheds we need a bound in the opposite direction. At $(\gamma, \tau) = (0, \tau_0)$ the acceptance probability stays away from one for small ρ_D: as $\rho_D \downarrow 0$, l'Hôpital gives $\pi_g \to (\theta_b\mu_s - \bar{x}_{P,b})/(\theta_b\mu_s - \bar{x}_{P,g})$, which is interior because $\bar{x}_{P,g} < \bar{x}_{P,b} < \theta_b\mu_s$. By continuity of π_g in ρ_D on the compact $[0,1]$, the map being defined on $(0,1]$ and extended continuously to $\rho_D = 0$ by the limit just computed, therefore, $\underline{\pi}_0 \equiv \inf_{\rho_D \in (0,1]} \big(1 - \pi_g(0, \tau_0)\big) > 0$. Hence the fully retained claim carries at least the trial-outcome risk

$$\mathrm{Var}_s(Y)\big|_{(0,\tau_0)} \;\geq\; \psi\, \underline{\pi}_0\, \theta_g^2 \sigma_0^2 \;\equiv\; \underline{V} \;>\; 0.$$

Now set

$$\tilde{\rho}_I \;\equiv\; \frac{\rho_P \underline{V}}{4\bar{B}} \qquad \text{and} \qquad \chi \;\equiv\; \frac{\bar{B}}{\psi \bar{B}_g} \;\geq\; 1,$$

where the inequality is immediate from $\bar{B} = \psi \bar{B}_g + \psi(1-\psi)\bar{M}^2$: the variance financing can shed onto the investor is never smaller than the variance the commitment effect adds to the defendant's exposure. Set

$$\tilde{\rho}_D \;\equiv\; \min\{1,\, \chi\, \tilde{\rho}_I\}.$$

The two restrictions are consistent with Assumption 2. Since $\chi \geq 1$, we have $\tilde{\rho}_D = \min\{1, \chi\tilde{\rho}_I\} \geq \min\{1, \tilde{\rho}_I\}$, so for any $\rho_I < \min\{1, \tilde{\rho}_I\}$ the interval $(\rho_I, \tilde{\rho}_D]$ of admissible defendant risk aversions is nonempty and the ordering $\rho_D > \rho_I$ can be maintained. For $\rho_I \leq \tilde{\rho}_I$ and $\rho_D \leq \tilde{\rho}_D$,

$$\begin{aligned}
\mathcal{R}(0,\tau_0) - \mathcal{R}(\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau_0), \tau_0) &= \tfrac{1}{2}\rho_P\, \mathrm{Var}_s(Y)\big|_{(0,\tau_0)} - \tfrac{1}{2} R(\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau_0))\, \mathrm{Var}_s(Y)\big|_{(\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau_0),\tau_0)} \\
&\quad + \tfrac{1}{2}\rho_D \psi\, \mathrm{Var}_s(Y \mid \theta_g)\big|_{(0,\tau_0)} - \tfrac{1}{2}\rho_D \psi\, \mathrm{Var}_s(Y \mid \theta_g)\big|_{(\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau_0),\tau_0)} \\
&\geq\; \tfrac{1}{2}\rho_P \underline{V} - \tfrac{1}{2}\rho_I \bar{B} - \tfrac{1}{2}\rho_D \psi \bar{B}_g \\
&\geq\; \tfrac{1}{2}\rho_P \underline{V} - \tfrac{1}{8}\rho_P \underline{V} - \tfrac{1}{8}\rho_P \underline{V} \\
&=\; \tfrac{1}{4}\rho_P \underline{V} \;>\; 0,
\end{aligned}$$

which is the result. The bound uses only that the private share lies above parity, and so holds at the private equilibrium share of every $\rho_I \leq \tilde{\rho}_I$.

Part (ii): Restrict $\theta_g \leq \theta_b/2$ and let

$$M_0(\gamma, \sigma_s^2) \;\equiv\; (\theta_b - \theta_g)\, \mu_s - \tfrac{1}{2}\, a_P(\gamma)\, \theta_b^2 \sigma_s^2$$

be the leading part of the transfer gap M, so that

$$\mathcal{R} \;=\; \tfrac{1}{2}\, \psi(1-\psi)\, R(\gamma)\, M_0(\gamma, \sigma_s^2)^2 + \theta_g^2 N(\gamma, \sigma_s^2),$$

where $\theta_g^2 N$ collects the trial-outcome and settlement-versus-trial terms, the defendant's premium, and the correction $M^2 - M_0^2 = (M - M_0)(M + M_0)$ with $M - M_0 = \tfrac{1}{2} a_P \sigma_s^2\, \pi_g \theta_g^2$. Every reservation is bilinear in (γ, σ_s^2) and π_g is a ratio of exponentials whose denominator is bounded below on the compact $(\gamma, \sigma_s^2) \in [0,1] \times [0, \sigma_0^2]$, since $\bar{x}_{D,b} - \bar{x}_{P,g} \geq (\theta_b -$

$\theta_g)\mu_s \geq \theta_b\mu_s/2$. Hence π_g and its partial derivatives are bounded there (the quotient rule places the same lower-bounded denominator, squared, under each derivative, while every numerator is a polynomial in exponentials of the reservations, all bounded on the compact uniformly in $\theta_g \leq \theta_b/2$), and since every collected term carries the factor θ_g^2, so are N, $\partial_{\sigma_s^2}N$, and $\partial_\gamma N$, bounded uniformly in $\theta_g \leq \theta_b/2$ by a constant K depending only on $(\theta_b, \mu_s, \rho_P, \rho_I, \rho_D, \psi, \sigma_0^2)$. Assumption 1, evaluated at $\sigma_s^2 = \sigma_0^2$, its most binding value, gives $\mu_s > \frac{1}{2}\rho_P(\theta_g + \theta_b)\sigma_0^2$, hence

$$M_0 \;\geq\; \theta_b\mu_s - \tfrac{1}{2}\rho_P\theta_b^2\sigma_0^2 - \theta_g\mu_s \;\geq\; \tfrac{1}{2}\big(\theta_b\mu_s - \tfrac{1}{2}\rho_P\theta_b^2\sigma_0^2\big) \;>\; 0 \quad \text{for } \theta_g \leq \frac{\theta_b\mu_s - \frac{1}{2}\rho_P\theta_b^2\sigma_0^2}{2\mu_s},$$

uniformly in (γ, σ_s^2). Assumption 1 bounds θ_g only from above, so the threshold constructed below may be taken as small as the argument requires.

On $\gamma \in [\gamma_{\text{rep}}^{\text{parity}}, 1]$ at the private informativeness, $\partial_\gamma M_0 = \frac{1}{2}\rho_P\theta_b^2\sigma_0^2 > 0$ and $R' \geq 0$ beyond parity give

$$\begin{aligned}
\partial_\gamma\mathcal{R}(\gamma, \sigma_0^2) \;&\geq\; \tfrac{1}{2}\psi(1-\psi)\Big[R'M_0^2 + 2R\,M_0\,\partial_\gamma M_0\Big] - \theta_g^2K \\
&\geq\; \underbrace{\tfrac{1}{4}\psi(1-\psi)R_{\min}\big(\theta_b\mu_s - \tfrac{1}{2}\rho_P\theta_b^2\sigma_0^2\big)\rho_P\theta_b^2\sigma_0^2}_{\equiv K_\gamma} - \theta_g^2K.
\end{aligned}$$

and at any share $\gamma \leq \bar{\gamma}$, $\partial_{\sigma_s^2}M_0 = -\frac{1}{2}a_P\theta_b^2$ and $a_P \geq (1-\bar{\gamma})\rho_P$ give

$$\begin{aligned}
\partial_{\sigma_s^2}\mathcal{R}(\gamma, \sigma_s^2) \;&\leq\; -\psi(1-\psi)R\,M_0\cdot\tfrac{1}{2}a_P\theta_b^2 + \theta_g^2K \\
&\leq\; -\underbrace{\tfrac{1}{4}\psi(1-\psi)R_{\min}\big(\theta_b\mu_s - \tfrac{1}{2}\rho_P\theta_b^2\sigma_0^2\big)(1-\bar{\gamma})\rho_P\theta_b^2}_{\equiv K_\sigma} \\
&\quad +\; \theta_g^2K.
\end{aligned}$$

Set

$$\underline{\theta}_g \;\equiv\; \min\Big\{\tfrac{1}{2}\theta_b,\; \frac{\theta_b\mu_s - \frac{1}{2}\rho_P\theta_b^2\sigma_0^2}{2\mu_s},\; \sqrt{\tfrac{K_\gamma}{2K}},\; \sqrt{\tfrac{K_\sigma}{2K}}\Big\}.$$

For $\theta_g < \underline{\theta}_g$ both gaps $K_\gamma - \theta_g^2K$ and $K_\sigma - \theta_g^2K$ are strictly positive. The cost of risk strictly increases in the share above parity at the private informativeness, and strictly decreases in σ_s^2 at any fixed share $\gamma \leq \bar{\gamma}$, equivalently, strictly increases in the informativeness τ on $[\tau_0, \infty)$, extending continuously to $\sigma_s^2 = 0$, the fully revealing price (the strict derivative bound in σ_s^2 on $(0, \sigma_0^2]$ delivers strict monotonicity on every bounded subinterval of $[\tau_0, \infty)$ and hence on the whole ray).

Write $\gamma_{\text{rep}} \equiv \gamma_{\text{rep}}^{\text{eq}}(\tau)$ for the public share, held fixed throughout what follows while the informativeness varies. For a benchmark level $\ell \in \mathbb{R}$ let

$$S(\ell;\gamma_{\mathrm{rep}}) \;\equiv\; \bigl\{\tau \in [\tau_0,\infty):\; \mathcal{R}(\gamma_{\mathrm{rep}},\tau) \le \ell\bigr\}.$$

Since $\mathcal{R}(\gamma_{\mathrm{rep}},\cdot)$ is continuous and strictly increasing in τ, by the derivative bound above together with the hypothesis $\gamma_{\mathrm{rep}} \le \bar{\gamma}$, $S(\ell;\gamma_{\mathrm{rep}})$ is a closed initial segment, $S(\ell;\gamma_{\mathrm{rep}}) = [\tau_0, \sup S(\ell;\gamma_{\mathrm{rep}})]$ whenever $\tau_0 \in S(\ell;\gamma_{\mathrm{rep}})$, and it shrinks in the benchmark, $S(\ell';\gamma_{\mathrm{rep}}) \subseteq S(\ell;\gamma_{\mathrm{rep}})$ for $\ell' \le \ell$. Take $\ell_P \equiv \mathcal{R}(\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau_0),\tau_0)$. By Proposition 2 and the ordering of the shares assumed in the statement, which Lemma 3 delivers, $\gamma^{\mathrm{parity}}_{\mathrm{rep}} < \gamma_{\mathrm{rep}} < \gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau_0)$, so monotonicity in the share gives $\mathcal{R}(\gamma_{\mathrm{rep}},\tau_0) < \ell_P$ and $\tau_0 \in S(\ell_P;\gamma_{\mathrm{rep}})$. Hence $S(\ell_P;\gamma_{\mathrm{rep}}) = [\tau_0,\bar{\tau}_{\mathcal{R}}]$ with $\bar{\tau}_{\mathcal{R}} \equiv \sup S(\ell_P;\gamma_{\mathrm{rep}}) \in [\tau_0,\infty]$, the claimed equivalence, and the inequality at τ_0 being strict, $\bar{\tau}_{\mathcal{R}} > \tau_0$ by continuity.

□

Proof of Proposition 7

Conditional on filing, the bad type settles with certainty and the good type's dispute reaches trial exactly upon rejection of its offer (Proposition 1), so $T(\gamma,\tau) = \psi\bigl(1-\pi_g(\gamma,\tau)\bigr)$ with the interior prior $\psi \in (0,1)$. The trial rate inherits the comparative statics of the acceptance probability with the sign reversed,

$$\partial_\gamma T = -\psi\,\partial_\gamma \pi_g \;>\; 0 \qquad \text{and} \qquad \partial_\tau T = -\psi\,\partial_\tau \pi_g \;>\; 0,$$

the first sign by Lemma 1 and the second by Lemma 2. A larger share and a more informative price each harden the separating demand and so push the good type toward trial.

For private markets the informativeness is held at τ_0 while the share rises from 0 to $\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau_0) > 0$, so $\partial_\gamma T > 0$ gives $T(\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau_0),\tau_0) > T(0,\tau_0)$ at once.

For public markets both arguments move, and by Proposition 4 they move as substitutes. The more informative price $\tau \ge \tau_0$ lowers the equilibrium share to $\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau) \le \gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau_0)$. That this substitution cannot reverse the comparison with retention follows by splitting the change into a share step at the public informativeness and an informativeness step at zero issuance,

$$T\bigl(\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau),\tau\bigr) - T(0,\tau_0) = \underbrace{\bigl[T(\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau),\tau) - T(0,\tau)\bigr]}_{\text{share step}} + \underbrace{\bigl[T(0,\tau) - T(0,\tau_0)\bigr]}_{\text{informativeness step}}.$$

The informativeness step is nonnegative by $\partial_\tau T \ge 0$. In the share step the equilibrium share survives the downward pull of the more informative price, $\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau) > 0$ by Proposition 2, so $\partial_\gamma T > 0$ makes it strictly positive. Hence $T(\gamma^{\mathrm{eq}}_{\mathrm{rep}}(\tau),\tau) > T(0,\tau_0)$. □

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